Preface

Providing an accurate quantitative description of wave propagation through a complex medium remains one of the more difficult mathematical problems of our age. In electrodynamics, many alternative-solution techniques for Maxwell's equations have emerged over the past century; all involve approximations in some form or another. These techniques range from approximations to the equations themselves, e.g., WKB, perturbation theory, parabolic equations, finite element, dynamic programming, scalar diffraction-based multiple phase-screen propagation using the Helmholtz–Kirkhoff theorem, etc., to straight ray theory techniques, with a good measure of hybrid wave/optics and spectral techniques included. Moreover, particularly when the wavelength of the wave is small compared to the scale of the refracting medium, one is forced to analytic and geometric simplifications to render such problems tractable. When one compares wave propagation literature from diverse fields, such as acoustics, electrodynamics, and seismology, one finds an almost bewildering variety of analysis techniques; one also finds, from time to time, essentially the same technique applied in different fields, but with different labels.

The radio occultation technique using spacecraft to sound a refracting medium is a relative newcomer in wave propagation processes and in the inverse problem of inferring certain physical properties of the medium. The first planetary occultation with spacecraft was accomplished about 40 years ago using the NASA/JPL Mariner 4 spacecraft as it passed behind Mars. The Mariner radio signal received on the Earth first transected the Martian ionosphere and atmosphere during its immersion before being eclipsed by Mars, and it passed through these media again after its egress from behind Mars. Using ray theory techniques, partly borrowed from seismology, these initial experiments successfully recovered accurate vertical refractivity profiles for the Martian atmosphere, and related density and pressure information.

These early ray theory techniques work well in a thin medium such as the Martian atmosphere and even in the upper reaches of other planetary atmospheres, such as the Earth's. But, as anyone who seriously studies sunrises and sunsets could predict, dense atmospheres with even simple mesocale layered structures, not to mention more complex structures, can lead to difficulty in the inversion process using basic ray theory techniques. Multiple rays from the transmitter following different paths through the atmosphere to the receiver, and atmosphere-induced broadening of the spectral lines of the individual rays, lead to ambiguity problems and analysis difficulties in a purely ray-theoretic approach. Moreover, ray theory, without recourse to at least an assist from scalar diffraction theory, imposes a certain resolution limit in the spatial dimensions perpendicular to the line of sight, the so-called first Fresnel zone, which is analogous to the Airy disk used in optical instruments to set resolution performance. Because of these limitations in basic ray theory, radio occultation investigators turned early on to wave/optics techniques to unravel multipath ambiguities and to sharpen the resolution using synthetic aperture concepts that approach the limit offered by wave theory. The thick atmospheres of the Jovian planets and the desire to sharpen the resolution of observations of the rings of these planets led to innovations in wave/optics techniques applied to the occultation experiments carried out with the Voyager spacecraft beginning about 25 years ago. The relatively recent advent of low Earth orbiters carrying occultation-capable GPS receivers (a half-dozen in the past decade and probably twice that many in the next decade) has resulted in a marked expansion in radio occultation science, in research and development in analysis techniques, and in technology development of new instrumentation and data information systems. The Earth's atmosphere is far more accessible than those of our solar system neighbors.

These modern inversion techniques, which on the whole have worked moderately well in adverse signal conditions such as those in the Earth's lower troposphere, may be classified, broadly speaking, as either spectral/holographic-based or as scalar diffraction-based. They all use some wave-related concept to recover the refractive bending angle(s) of the "received" ray(s), and through a certain ray-theoretic inversion process, they recover the vertical refractivity profile of the atmosphere. This hybrid wave-ray methodology is often referred to as a "wave/optic" analysis process.

This monograph develops, as its central theme, a purely wave-theoretic approach. The amplitude and phase of a harmonic wave at a given point are formally expressed in terms of a complex spectral series, the spectral coefficients of which depend on the refractive gradient of the medium in addition to the spectral number and the initial conditions for the wave. Therefore, the spectral coefficients vary with position in the medium because of the non-zero refractive gradient there, whereas in the homogeneous medium they are invariant. The total contribution to the electromagnetic field at a given point is obtained by summing this spectral series over all spectral numbers. Stationary phase points in spectral number signify the major neighborhoods contributing to the evaluation of the field. The goal here is to recover the spectral coefficients from the time series of observed amplitude and phase of the received wave. The magnitude of a spectral coefficient relates to the absorption by the atmosphere; its phase shift relative to the constant phase that would hold in a homogeneous medium relates to the cumulative effect of the refractive gradient of the atmosphere on that spectral component of the wave. The phase shift for the spectral coefficient is given by a spectral density function for refractive gradient-induced phase delay. From the recovery of this spectral density function, the profile for the refractivity gradient is obtained from the formal integral relationship between these quantities. Although full-spectrum wave theory approaches for these kinds of problems are notorious for their complexity and cumbersomeness, it nevertheless is possible to render the mathematics into a reasonably tractable form through judicious use of asymptotic techniques, stationary phase concepts, and so on. Special topics familiar in ray theory, such as rays, caustics, reflections, defocusing, multipath, and so on, are treated here in a wave-theoretic framework.

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April 2004