Chapter 5
Single-Receiver Performance

In this chapter, the performance characteristics of a single receiver are derived in such a way that the parameters defining this performance can be carried over to an array, allowing comparison between the various arraying techniques.

5.1 Basic Equations

In deep-space communications, the downlink symbols first are modulated onto a square-wave subcarrier, and then the modulated subcarrier is modulated onto an RF carrier [1]. This allows transmission of a residual-carrier component whose frequency does not coincide with the data spectrum and, therefore, minimizes interference between the two. At the receiver, the deep-space signal is demodulated using a carrier-tracking loop, a subcarrier-tracking loop [2], and a symbol-synchronizer loop [3], as shown in Fig. 5-1. Depending on the modulation index, carrier tracking can be achieved by a phase-locked loop (PLL), Costas loop, or both [4]. The PLL or a combination of loops is used for modulation indices less than 90 deg, whereas a Costas loop is used when the modulation index is 90 deg. The received signal from a deep-space spacecraft can be modeled as

$$r(t) = s(t) + n(t)$$

where

$$\begin{aligned} s(t) &= \sqrt{2P}\sin\left[\omega_c t + \Delta\, d(t)\, Sqr\left(\omega_{sc} t + \theta_{sc}\right) + \theta_c\right] \\ &= \sqrt{2P_c}\sin\left(\omega_c t + \theta_c\right) + \sqrt{2P_d}\, d(t)\, Sqr\left(\omega_{sc} t + \theta_{sc}\right)\cos\left(\omega_c t + \theta_c\right) \end{aligned} \quad (5.1\text{-}1)$$

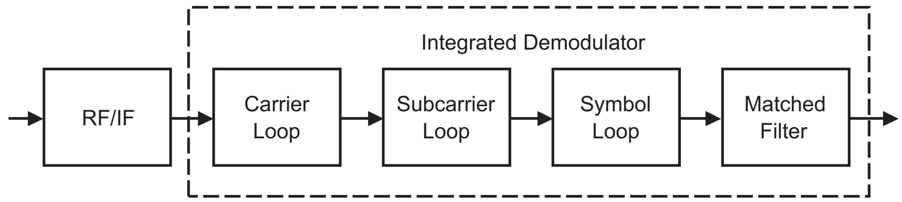

Fig. 5-1. A general coherent receiver model.

The carrier and data powers, denoted P_c and P_d, are given by $P\cos^2\Delta$ and $P\sin^2\Delta$, respectively, and P is the total received signal power, Δ is the modulation index, ω_c and θ_c are the carrier frequency and phase, $n(t)$ is an additive bandlimited white Gaussian noise process, $d(t)$ is the nonreturn-to-zero (NRZ) or Manchester data, and $Sqr(\ )$ designates the square-wave subcarrier with frequency ω_{sc} and phase θ_{sc}. Here the first component, the residual carrier, typically is tracked by a phase-locked loop, and the second component, the suppressed carrier, can be tracked by a Costas loop. The modulation $d(t)$ is given by

$$d(t) = \sum_{k=-\infty}^{\infty} d_k\, p(t - kT_s) \tag{5.1-2}$$

where d_k is the ± 1 binary data, T_s is the symbol period, and $p(t)$ is a baseband pulse of unit power and limited to T_s seconds. The narrowband noise $n(t)$ can be written as

$$n(t) = \sqrt{2}n_c(t)\cos(\omega_c t + \theta_c) - \sqrt{2}n_s(t)\sin(\omega_c t + \theta_c) \tag{5.1-3}$$

where $n_c(t)$ and $n_s(t)$ are statistically independent, stationary, bandlimited white Gaussian noise processes with one-sided spectral density level N_0 (W/Hz) and one-sided bandwidth B (Hz), which is large compared to $1/T_s$.

The primary function of a receiver is to coherently detect the transmitted symbols as illustrated in Fig. 5-1. The demodulation process requires carrier, subcarrier, and symbol synchronization. The output of the receiver, v_k, is derived in Appendix C and given by

$$v_k = \sqrt{P_d}\, C_c\, C_{sc}\, C_{sy}\, d_k + n_k \tag{5.1-4}$$

where C_c, C_{sc}, and C_{sy} denote the carrier-, subcarrier-, and symbol-reduction functions and are given by

$$
\begin{aligned}
C_c &= \cos\phi_c \\
C_{sc} &= \left(1 - \frac{2}{\pi} |\phi_{sc}|\right) \\
C_{sy} &= \left(1 - \frac{1}{2\pi} |\phi_{sy}|\right)
\end{aligned}
\tag{5.1-5}
$$

and ϕ_c, ϕ_{sc}, and ϕ_{sy} denote carrier, subcarrier, and symbol phase errors, respectively, and n_k is a Gaussian random variable with variance $\sigma_n^2 = N_0/2T_s$. Symbol SNR degradation is defined as the average reduction in SNR at the symbol matched-filter output due to imperfect synchronization, whether carrier, subcarrier, or symbol. Ideally, $\phi_c = \phi_{sc} = \phi_{sy} = 0$, and Eq. (5.1-4) reduces to the ideal matched-filter output $v_k = \sqrt{P} d_k + n_k$, as expected. In deriving Eq. (5.1-4), it is assumed that the carrier, subcarrier, and symbol-loop bandwidths are much smaller than the symbol rate so that the phase errors ϕ_c, ϕ_{sc}, and ϕ_{sy} can be assumed to be constant over several symbols. Throughout this chapter, ϕ_c is assumed to be Tikhonov distributed [5]:

$$
p(\phi_c) = \frac{e^{\rho_c \cos\phi_c}}{2\pi I_0(\rho_c)}, \quad |\phi_c| \le \pi \tag{5.1-6}
$$

and ϕ_{sc} and ϕ_{sy} are assumed to be Gaussian distributed, i.e.,

$$
p(\phi_i) = \frac{e^{-\phi_i^2/2\sigma_i^2}}{\sqrt{2\pi\sigma_i^2}}, \qquad i = sc, sy \tag{5.1-7}
$$

where $\rho_i = 1/\sigma_i^2$ denotes the respective loop SNR and *p*( ) is a probability density function.

5.2 Degradation and Loss

A useful quantity needed to compute degradation and loss is the symbol SNR conditioned on ϕ_c, ϕ_{sc}, and ϕ_{sy}. The conditional symbol SNR, denoted SNR′, is defined as the square of the conditional mean computed with respect to the thermal noise of v_k divided by the conditional variance of v_k, i.e.,

$$
\text{SNR}' = \frac{\left[\overline{(v_k | \phi_c, \phi_{sc}, \phi_{sy})}\right]^2}{\sigma_n^2} = \frac{2P_d T_s}{N_0} C_c^2 C_{sc}^2 C_{sy}^2 \tag{5.2-1}
$$

where $\overline{(x|y)}$ denotes the statistical expectation of *x* conditioned on *y*, and v_k and σ_n^2 are as defined earlier.

The unconditional signal-to-noise ratio, denoted SNR, is found by averaging Eq. (5.2-1) over the carrier, subcarrier, and symbol phases. Letting $\overline{x}$ denote the average of *x*, the unconditional SNR is given as

$$\text{SNR} = \frac{2P_d T_s}{N_0} \overline{C_c^2}\, \overline{C_{sc}^2}\, \overline{C_{sy}^2} \tag{5.2-2}$$

Ideally, when there are no phase errors (i.e., when $\phi_c = \phi_{sc} = \phi_{sy} = 0$), $C_c = C_{sc} = C_{sy} = 1$ and Eq. (5.2-2) reduces to $\text{SNR}_{\text{ideal}} = 2P_d T_s / N_0$, as expected. The symbol SNR degradation, D, is defined as the ratio of the unconditional SNR at the output of the matched filter in the presence of imperfect synchronization to the ideal matched-filter output SNR. The degradation, *D*, in dB for a single antenna thus is given by

$$D = 10\log_{10}\left[\frac{\text{SNR}}{\text{SNR}_{\text{ideal}}}\right] = 10\log_{10}\left(\overline{C_c^2}\, \overline{C_{sc}^2}\, \overline{C_{sy}^2}\right) \tag{5.2-3}$$

Before proceeding, we need to understand and quantify the degradations due to the carrier, subcarrier, and symbol synchronization. Carrier tracking can be performed in two ways. The residual component of the signal can be tracked with a phase-locked loop or the suppressed component of the signal can be tracked with a Costas loop (see Sections 3.2 and 3.3 in [5]). With a PLL, the loop SNR is given by

$$\rho_{c,r} = \frac{1}{\sigma_{c,r}^2} = \frac{P_c}{N_0 B_c} \tag{5.2-4}$$

where B_c is the carrier-loop bandwidth and $\sigma_{c,r}^2$ is the phase jitter in the loop (the subscript "*c,r*" refers to the carrier residual component). On the other hand, with a Costas loop, we have

$$\rho_{c,s} = \frac{1}{\sigma_{c,s}^2} = \frac{P_d\, S_L}{N_0\, B_c} \tag{5.2-5}$$

where S_L is the squaring loss given by $S_L^{-1} = 1 + \left(1/\left[2E_s / N_0\right]\right)$, and $E_s / N_0 = P_d T_s / N_0$ is the symbol SNR (the subscript "*c,s*" refers to the carrier suppressed component). Note from Eq. (5.1-1) that, when $\Delta = 90$ deg, the residual component disappears, and the carrier is fully suppressed. On the other hand, when $\Delta = 0$ deg, the signal reduces to a pure sine wave. When Δ is not exactly 0 or 90 deg, both components of the carrier (residual and suppressed) can be tracked simultaneously, and the carrier phase estimates can be combined to provide an improved estimate. This is referred to as sideband aiding (SA), and it results in an improved carrier-loop SNR given to a first-order approximation by

$$\rho_c = \rho_{c,r} + \rho_{c,s} \tag{5.2-6}$$

Whether sideband aiding is employed or not, the degradation due to imperfect carrier reference is given by $\overline{C_c^2}$.

The subcarrier-loop phase jitter, σ_{sc}^2, in a Costas loop is given by [4]

$$\sigma_{sc}^2 = \frac{1}{\rho_{sc}} = \left(\frac{\pi}{2}\right)^2 \frac{B_{sc} w_{sc}}{P_d / N_0} \left(1 + \frac{1}{2 E_s / N_0}\right) \tag{5.2-7}$$

where w_{sc} denotes the subcarrier window. Similarly, the symbol-loop phase jitter, σ_{sy}^2, assuming a data-transition tracking loop (DTTL), is [5]

$$\sigma_{sy}^2 = \frac{1}{\rho_{sy}} = \frac{2\pi^2 B_{sy} w_{sy}}{\left(P_d / N_0\right) erf^2\left(\sqrt{E_s / N_0}\right)} \tag{5.2-8}$$

where w_{sy} is the symbol window and $erf(\cdot)$ denotes the error function. The probability density functions (pdfs) of ϕ_c and ϕ_{sy} can be assumed to be Gaussian or Tikhonov. Assigning a Tikhonov density for the carrier phase error and a Gaussian density for the other two, the first two moments of C_c, C_{sc}, and C_{sy} of Eq. (5.1-5) become, respectively,

$$\begin{aligned}
\overline{C_c} &= \overline{\cos\phi_c} = \frac{I_1(\rho_c)}{I_0(\rho_c)} \\
\overline{C_c^2} &= \overline{\cos^2\phi_c} = \frac{1}{2}\left[1 + \frac{I_2(\rho_c)}{I_0(\rho_c)}\right] \\
\overline{C_{sc}} &= 1 - \frac{2}{\pi}\overline{|\phi_{sc}|} = 1 - \left(\frac{2}{\pi}\right)^{3/2}\sigma_{sc} \\
\overline{C_{sc}^2} &= 1 - \frac{4}{\pi}\overline{|\phi_{sc}|} + \frac{4}{\pi^2}\overline{\phi_{sc}^2} = 1 - \sqrt{\frac{32}{\pi^3}}\sigma_{sc} + \left(\frac{2}{\pi}\right)^2\sigma_{sc}^2 \\
\overline{C_{sy}} &= 1 - \frac{1}{2\pi}\overline{|\phi_{sy}|} = 1 - \sqrt{\frac{1}{2\pi}}\frac{\sigma_{sy}}{\pi} \\
\overline{C_{sy}^2} &= 1 - \frac{1}{\pi}\overline{|\phi_{sy}|} + \frac{1}{4\pi^2}\overline{\phi_{sy}^2} = 1 - \sqrt{\frac{2}{\pi}}\frac{\sigma_{sy}}{\pi} + \frac{\sigma_{sy}^2}{4\pi^2}
\end{aligned} \tag{5.2-9}$$

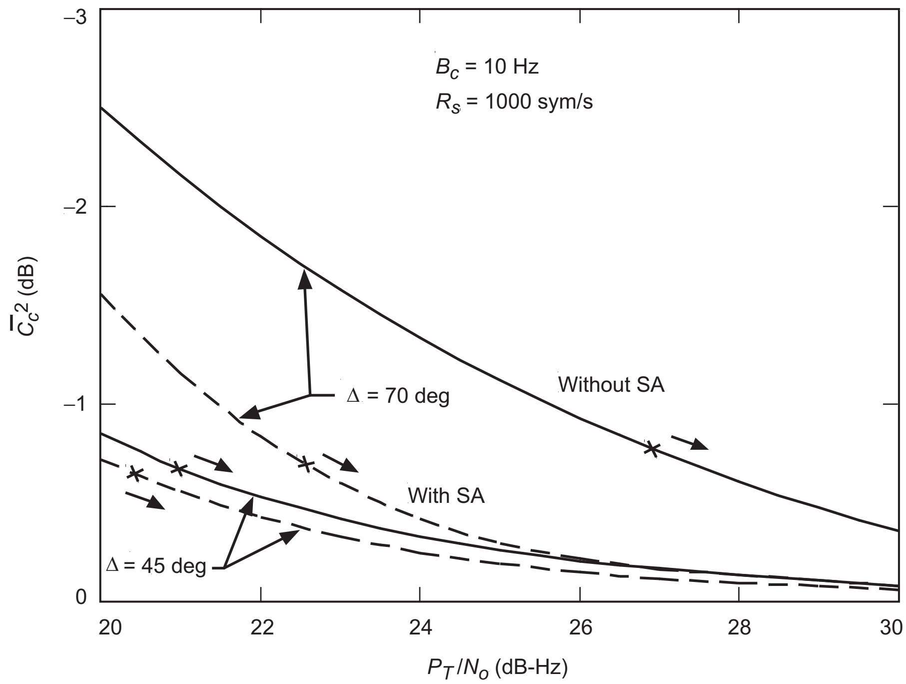

Fig. 5-2. Symbol SNR degradation due to imperfect carrier reference.

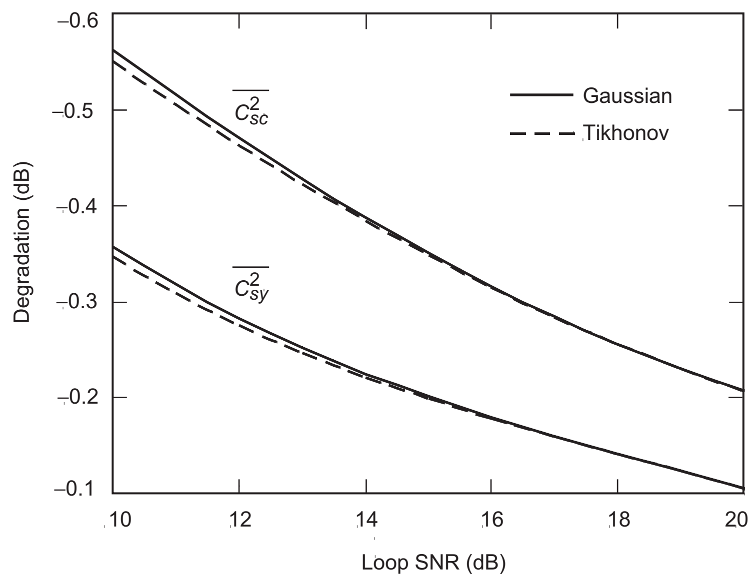

Fig. 5-3. Symbol SNR degradation in the presence of subcarrier and symbol phase jitter.

where $I_k(\cdot)$ denotes the modified Bessel function of order k and ρ_c is the carrier-loop SNR. The symbol degradations, $\overline{C_c^2}, \overline{C_{sc}^2}$, and $\overline{C_{sy}^2}$, versus the loop SNR are depicted in Figs. 5-2 and 5-3. Figure 5-3 also depicts the degradation, assuming ϕ_{sc} and ϕ_{sy} are either Gaussian or Tikhonov distributed. It is clear from this figure that both densities provide close results; therefore, the Gaussian assumption for the subcarrier and symbol phase errors will be utilized from here on.

The notion of loss is defined in terms of the desired bit- or symbol-error rate (SER). For the single receiver shown in Fig. 5-1, the symbol error rate, denoted $P_s(E)$, is defined as

$$P_s(E) = \iiint P_s\left(E \mid \frac{E_s}{N_0}, \phi_c, \phi_{sc}, \phi_{sy}\right) p_c(\phi_c) p_{sc}(\phi_{sc}) p_{sy}(\phi_{sy})\, d\phi_c d\phi_{sc} d\phi_{sy} \quad (5.2\text{-}10)$$

where $P_s\left(E \mid [E_s / N_0], \phi_c, \phi_{sc}, \phi_{sy}\right)$ is the symbol-error rate conditioned on the symbol SNR and on the phase errors in the tracking loops. For the uncoded channel,

$$P_s\left(E \mid \frac{E_s}{N_0}, \phi_c, \phi_{sc}, \phi_{sy}\right) = \frac{1}{2} erfc\left[\sqrt{\frac{E_s}{N_0} C_c^2 C_{sc}^2 C_{sy}^2}\right] \quad (5.2\text{-}11)$$

where $erfc(\cdot)$ is the complementary error function. Ideally, when there are no phase errors (i.e., when $\rho_c = \rho_{sc} = \rho_{sy} = \infty$, so that $C_c = C_{sc} = C_{sy} = 1$), Eq. (5.2-11) reduces to the well-known binary phase-shift keyed (BPSK) error rate,

$$P_s(E) = \frac{1}{2} erfc\left[\sqrt{\frac{E_S}{N_0}}\right] \quad (5.2\text{-}12)$$

Symbol SNR loss is defined as the additional symbol SNR needed in the presence of imperfect synchronization to achieve the same SER as in the presence of perfect synchronization. Mathematically, the SNR loss due to imperfect carrier-, subcarrier-, and symbol-timing references is given in dB as

$$L = 20\log_{10}\left[erfc^{-1}\left(2P_{s\ \mathrm{ideal}}(E)\right)\right] - 20\log_{10}\left[erfc^{-1}\left(2P_{s\ \mathrm{actual}}(E)\right)\right] \quad (5.2\text{-}13)$$

The first term in the above equation is the symbol SNR required for a given symbol-error rate in the presence of perfect synchronization, whereas the second term is the symbol SNR required with imperfect synchronization.

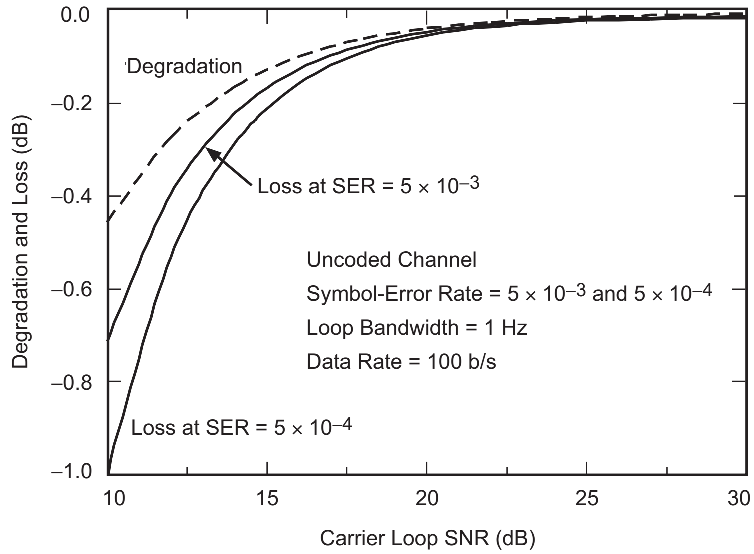

Fig. 5-4. Degradation and loss versus carrier-loop SNR.

Figure 5-4 depicts degradation and loss curves for the carrier loop. Note that loss is a function of $P_s(E)$, while degradation is not. Also, loss provides a more accurate performance prediction at the expense of added computational complexity and should be used when possible.

References

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