Chapter 8
Correlation Algorithms

There are several algorithms that possibly can be employed in those circumstances when correlation of signals is the method used to determine the phase and delay offsets between the array antennas (FSC, CSC, BA). If the SNR of the signal from each antenna is high enough to obtain a strong correlation for all the antenna pairs, then no special processing is necessary, and the phase and delay offsets derived from the correlation can be used directly to align the signals. However, when the signal SNR from each antenna is low, usually some other approach is necessary to take advantage of all possible antenna pairs. In this chapter, we discuss several approaches that have been analyzed and implemented with the arrays used in the DSN, including some discussion of their relative merits.

8.1 General

The output of an array is a weighted sum of the input signals applied to the combiner, where each of these input signals comes from the various antennas in the array. Here we assume that the input signals to the combiner have been corrected using predicts so that the residual delay and phase between the signals are slowly varying. The complex weights, providing corrections for both the amplitude and phase of the signals, can be derived in a number of ways from the cross-correlation matrices of the signal plus noise and of the noise itself. These matrices are derived by summing each combiner input over a symbol length, multiplying the sums from each pair of antennas, and accumulating long enough to obtain an adequately high signal-to-noise ratio. An array of L antennas will yield an $L \times L$ hermitian matrix of correlation components. The length of time over which the elements of the matrices can be accumulated is limited mostly by phase variations in the input signals caused by the troposphere (instrumental phase variations usually are much smaller). A noise cross-correlation matrix can be obtained by moving off the signal source either spatially (for broadband sources) or spectrally (for narrowband sources such as spacecraft). In general, the amplitude of the weights is proportional to the input signal voltage divided by its variance, which takes into account both the signal-to-noise ratio and gain of the signals [1].

In an actual implementation, the delay and phase corrections are applied to the antenna signals before correlation in the form of a locking loop for each of these parameters. The purpose of the loops is to drive the delay and phase residuals to zero. Since delay is expected to vary much more slowly compared to phase, the order of the delay loop is smaller and its bandwidth is narrower. The phase-locked loop is second order and typically uses a 0.1-Hz bandwidth (10-s integration). The delay-locked loop is first order and typically uses a 0.01-Hz bandwidth (100-s integration). In addition, a history of the delay residuals is accumulated and used in such a way as to allow even narrower effective bandwidths (longer integrations) for this loop, permitting the delay residuals to be even more well-determined.

8.2 Simple

The Simple algorithm is diagramed in Fig. 8-1. One of the antennas in an array of L elements is designated as the reference antenna. This usually is the antenna with the largest G/T, although this is not an absolute requirement. Since the reference antenna becomes the phase center for the array, one may have reason to choose another antenna for this role. The signal from each of the remaining $(L - 1)$ antennas is then correlated with the signal from this reference antenna to yield $(L - 1)$ complex correlation amplitudes. This corresponds to one row of the correlation matrix mentioned above and is simple to implement since the amount of processing needed is proportional to the number of antennas. These complex amplitudes are used to correct the individual antenna signals to bring them into phase and delay coherence with the reference antenna signal. The resulting L signals then can be added to give an improvement in SNR. The improvement will depend on how well the corrected signals line up in phase. Limitation on the accuracy of the correction phases is determined by the averaging time that can be used in obtaining the correlation amplitudes. As mentioned above, this averaging time is largely restricted by phase variations in the antenna signals due to their passing through the troposphere

8.3 Sumple

The Sumple method is diagramed in Fig. 8-2. It can be described as the cross-correlation of each antenna with a reference antenna composed of the weighted sum of all the other antennas. It is an iterative method and can be used

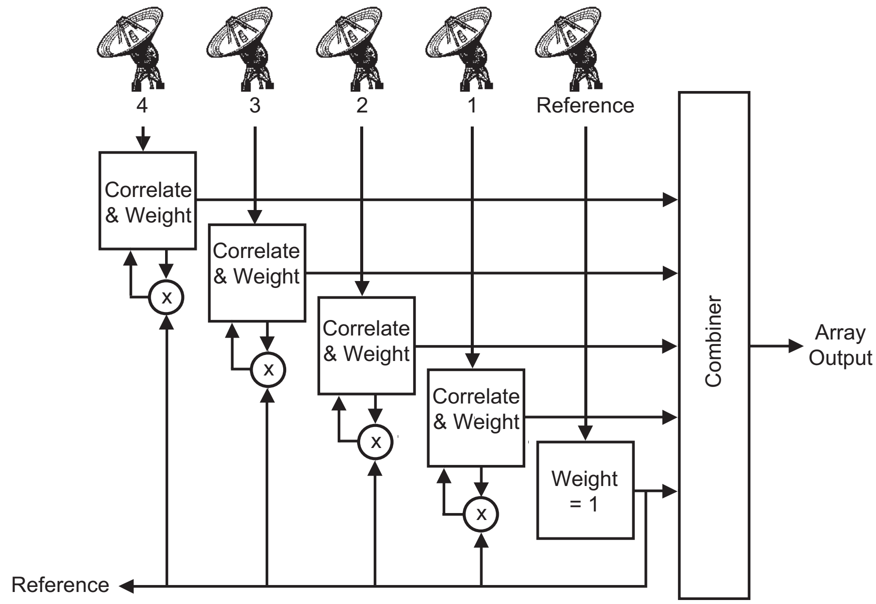

Fig. 8-1. Diagram for the Simple method of combining.

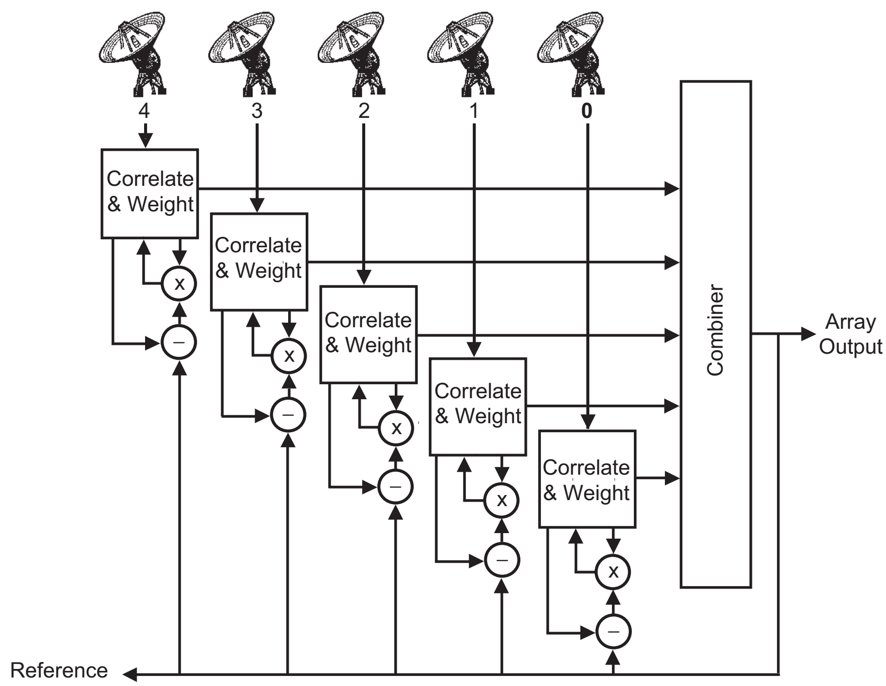

Fig. 8-2. Diagram for the Sumple method of combining.

for weaker signals than can the Simple method. The processing required is more complicated than the Simple method, but it is still proportional to the number of antennas. A single iteration is accomplished by rotating through each of the L antennas, correlating it with the complex weighted sum of the remaining antennas. The weights begin as unit vectors of zero phase. After each iteration, the previous weights are replaced with the new weights, and the process is repeated. The method appears to always converge from a random state in a few iterations (between 4 and 8), primarily because the weighted sum of all the antennas (minus one) is nearly a constant vector (albeit initially of small amplitude for a large numbers of antennas). Unlike the Simple method, the various antennas are not brought into alignment with a single reference antenna, but instead line up to a kind of floating reference. Simulations suggest the phase wandering of this reference is larger at lower SNR, but never becomes much more than a fraction of a cycle per hour, somewhat smaller than other sources of phase instability.

8.4 Eigen

The Eigen method of deriving the complex weights is given in [2] and uses both the signal matrix and the noise matrix mentioned above. It appears to be very general. The amount of processing required is proportional to the number of antennas squared, but the method does take into account off-diagonal noise coming from, for example, a background planet. In this case, the complex weights will maximize the SNR of the combined signal by a blend of aligning the phases of the desired signals and de-aligning the phases of the interfering signal (the planet).

8.5 Least-Squares

The Least-Squares method takes advantage of the fact that for an array of N antennas there are only $N - 1$ unknown relative phases and $N(N - 1)/2$ independent measurements in the correlation matix. The $N - 1$ phases are adjusted using an iterative procedure to minimize the difference between the predicted and measured cross-correlation matrices. Like the Eigen method, the processing is proportional to the square of the number of antennas.

8.6 Simulations

While only the Simple and Sumple algorithms were actually implemented in the 34-m arraying system, all four methods were simulated in a general-purpose computer. Of greatest interest is their performance at low SNR with various numbers of antennas. Figure 8-3 gives some of the results. As can be seen from the diagram, all give similar losses when the loop integration time is long. For short integration times, the Sumple algorithm performs best, but is

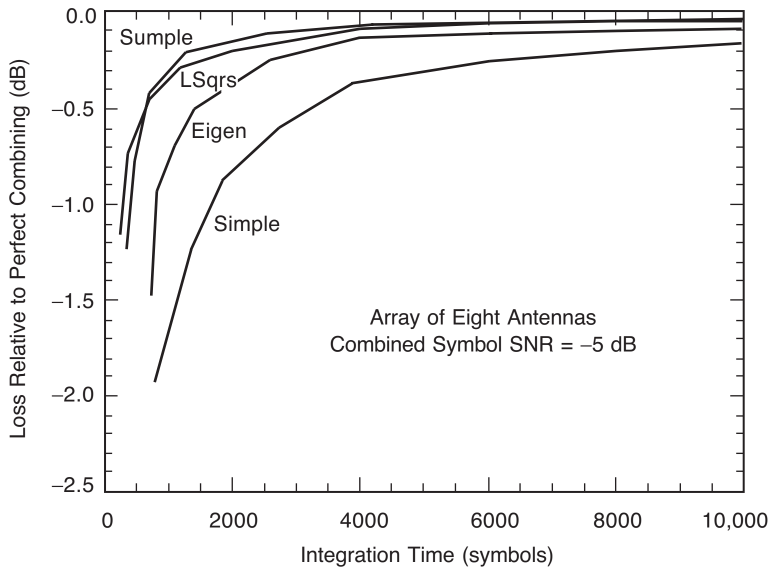

Fig. 8-3. Simulation of combining loss verses loop integration time.

very similar to the Least-Squares method. What is surprising is that Sumple is better than Eigen, even though it also uses all of the correlation pairs. One guess as to the reason for this difference in performance is that Eigen, being more general, is less constrained and therefore more sensitive to noise.

References

[1] V. Vilnrotter and E. R. Rodemich, "Real-Time Combining of Residual Carrier Array Signals Using ML Weight Estimates," *IEEE Trans. Comm.*, vol. COM-40, no. 3, pp. 604–615, March 1992.

[2] K .-M. Cheung, "Eigen Theory for Optimal Signal Combining: A Unified Approach," *The Telecommunications and Data Acquisition Progress Report 42-126, April–June 1996*, Jet Propulsion Laboratory, Pasadena, California, pp. 1–9, August 15, 1996.

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