Appendix G
Derivation of Equations for Complex-Symbol Combining

G.1 Derivation of Eq. (6.2-5)

Substituting Eq. (6.2-2) into Eq. (6.2-4), one obtains

$$\mathbf{v}_k = \sum_{i=1}^{L} \beta_i \left(\sqrt{P_i}\, C_{sc\,i} C_{sy\,i} d_k e^{j[\Delta\omega_c t_k + \Delta\phi_{i1}]} + \mathbf{n}_{k\,i} e^{-j\hat{\theta}_{i1}} \right) \tag{G-1}$$

where $\Delta\phi_{i1} = \theta_{i1} - \hat{\theta}_{i1}$ and all other symbols are defined in Eq. (6.2-1). The conditional combined power, denoted P', in Eq. (6.2-5) is found by deriving the conditional mean of $\mathbf{v}_k$, i.e.,

$$\begin{aligned} P' &= \mathrm{E}\left[\mathbf{v}_k \mid \phi_{sc_i}, \phi_{sy_i}, \Delta\phi_{i1} \right] \mathrm{E}^* \left[\mathbf{v}_k \mid \phi_{sc_j}, \phi_{sy_j}, \Delta\phi_{j1} \right] \\ &= \sum_{i=1}^{L} \sum_{j=1}^{L} \beta_i \beta_j \sqrt{P_i} \sqrt{P_j} C_{sc\,i} C_{sy\,i} C_{sc\,j} C_{sy\,j} e^{j[\Delta\phi_{i1} + \Delta\phi_{j1}]} \end{aligned} \tag{G-2}$$

which simplifies to Eq. (6.2-7). In addition, the phase $\theta_{\mathbf{v}}$ in Eq. (6.2-5) is given as

$$\theta_{\mathbf{v}} = \tan^{-1}\left[\frac{\Im\left[\sum_{i=1}^{L}\beta_i\sqrt{P_i}\,C_{sc\,i}C_{sy\,i}e^{j[\Delta\omega_c t_k+\Delta\phi_{i1}]}\right]}{\Re\left[\sum_{i=1}^{L}\beta_i\sqrt{P_i}\,C_{sc\,i}C_{sy\,i}e^{j[\Delta\omega_c t_k+\Delta\phi_{i1}]}\right]}\right] \tag{G-3}$$

G.2 Derivation of Eq. (6.2-11)

Let $C_{sy\,i}$ be the signal-reduction function due to symbol-timing errors in the ith symbol-synchronization loop. Then the ith matched-filter output in Eq. (6.2-2) can be rewritten as

$$\mathbf{v}_{ki} = \sqrt{P_i}C_{sc\,i}C_{sy\,i}d_k e^{j[\Delta\omega_c t_k+\theta_{i1}]} + \mathbf{n}_{ki} \tag{G-4}$$

The relative phase difference between antenna i and the reference antenna is estimated by performing the correlation operation shown in Fig. 6-8. Assuming perfect time alignment, the correlation output, $\mathbf{v}$, is given as

$$\mathbf{v} = \sum_{k=1}^{N}\mathbf{v}_{ki}\mathbf{v}_{k1}^{*} \tag{G-5}$$

where $N = T/T_s$ is the number of symbols used in the correlation. The correlation time and symbol time are denoted as T and T_s, respectively. Substituting the expressions for $\mathbf{v}_{ki}$ and $\mathbf{v}_{k1}$ into Eq. (G-5) (the performance of the full-spectrum correlator) yields

$$\mathbf{v} = \sqrt{P_1P_i}C_{sc1}C_{sy\,i}C_{sc1}C_{sy\,i}e^{j\theta_{i1}} + \mathbf{n}_{\mathbf{v}} \tag{G-6}$$

where

$$\mathrm{Var}(\mathbf{n}_{\mathbf{v}}) = 2P_1\overline{C_{sc1}^2}\,\overline{C_{sy1}^2}\frac{N_{0i}}{2T} + 2P_i\overline{C_{sc\,i}^2}\,\overline{C_{sy\,i}^2}\frac{N_{01}}{2T} + 2\frac{N_{01}N_{0i}}{2T_sT} \tag{G-7}$$

Defining the SNR for complex signals as $\mathrm{SNR} = E(\mathbf{v})E(\mathbf{v}^*)/Var(\mathbf{v})$, the correlator SNR between antenna i and antenna 1 for CSC is given as

$$\mathrm{SNR}_{csci1} = \frac{\sqrt{P_1P_i}\,\overline{C}_{sc1}\overline{C}_{sc\,i}\overline{C}_{sy1}\overline{C}_{sy\,i}}{P_1\overline{C_{sc1}^2}\,\overline{C_{sy1}^2}\frac{N_{0i}}{T} + P_i\overline{C_{sc\,i}^2}\,\overline{C_{sy\,i}^2}\frac{N_{01}}{T} + \frac{N_{01}N_{0i}}{T_sT}} \tag{G-8}$$

and simplifying yields Eq. (6.2-11).