Appendix E
Closed-Loop Performance

Typically, one would like to limit the IF combining losses expressed by Eq. (6.1-31) to some pre-specified maximum value, say $D_{\max}$. Solving Eq. (6.1-31) for $\sigma^2_{\Delta\phi\ \max}$, we obtain

$$\sigma^2_{\Delta\phi\ \max} < -2\ln\left[\frac{10^{D_{\max}/10}L-1}{L-1}\right] \tag{E-1}$$

The variance of the phase estimate can be reduced either by increasing the correlation time T in Eq. (6.1-12) or by tracking the phase-error process in a closed-loop fashion. Note that the value of B in Eq. (6.1-12) is set by the bandwidth of the telemetry spectrum and cannot be reduced at will.

In the simplest closed-loop implementation of the full-spectrum combining scheme, phase-error estimates can be updated using the following difference equation:

$$\hat{\theta}(n) = \hat{\theta}(n+1) + \alpha\,\phi(n) \tag{E-2}$$

where the value of α can be set between 0.2 and 0.5, and $\hat{\theta}(n)$ is the filtered phase-error estimate. The above difference equation gives the following loop transfer function:

$$G(z) = \frac{\hat{\Theta}(z)}{\Phi(z)} = \frac{\alpha}{z-1} \tag{E-3}$$

The variance of the closed-loop phase-error process now will be

$$\sigma_{i1}^2 = I_1 \sigma_{\Delta\phi\, i1}^2 = I_1 \frac{N_{01} N_{oi} B}{2 P_1 P_i T} \tag{E-4}$$

where

$$I_1 = \frac{1}{2\pi j} \oint |H(z)|^2 \frac{dz}{z} \tag{E-5}$$

and $H(z) = G(z)/(1 + G(z))$. Using the above $G(z)$, we obtain

$$I_1 = \frac{\alpha}{2 - \alpha} \tag{E-6}$$

As an example, for $\alpha = 0.2$, $I_1 = 0.11$, and the variance of the phase jitter is reduced by a factor of ten.