Appendix E
Closed-Loop Performance

Typically, one would like to limit the IF combining losses expressed by Eq. (6.1-31) to some pre-specified maximum value, say $D_{\text{max}}$. Solving Eq. (6.1-31) for $\sigma_{\Delta \phi}^2$, we obtain

$$\sigma_{\Delta \phi}^2 < -2 \ln \left[ \frac{10^{D_{\text{max}}/10} L - 1}{L - 1} \right]$$

The variance of the phase estimate can be reduced either by increasing the correlation time $T$ in Eq. (6.1-12) or by tracking the phase-error process in a closed-loop fashion. Note that the value of $B$ in Eq. (6.1-12) is set by the bandwidth of the telemetry spectrum and cannot be reduced at will.

In the simplest closed-loop implementation of the full-spectrum combining scheme, phase-error estimates can be updated using the following difference equation:

$$\hat{\theta}(n) = \hat{\theta}(n + 1) + \alpha \phi(n)$$

where the value of $\alpha$ can be set between 0.2 and 0.5, and $\hat{\theta}(n)$ is the filtered phase-error estimate. The above difference equation gives the following loop transfer function:

$$G(z) = \frac{\hat{\Theta}(z)}{\Phi(z)} = \frac{\alpha}{z - 1}$$

The variance of the closed-loop phase-error process now will be
where

\[ I_1 = \frac{1}{2\pi j} \oint |H(z)|^2 \frac{dz}{z} \quad (E-5) \]

and \( H(z) = G(z)/(1 + G(z)) \). Using the above \( G(z) \), we obtain

\[ I_1 = \frac{\alpha}{2 - \alpha} \quad (E-6) \]

As an example, for \( \alpha = 0.2 \), \( I_1 = 0.11 \), and the variance of the phase jitter is reduced by a factor of ten.