Appendix C
Demodulation Process

C.1 Signal Model

Carrying over Eq. (5.1-1) from Chapter 5 with some simplification, and assuming the signal has been digitally sampled, the received signal from a deep-space spacecraft can be modeled as

$$\begin{aligned} s_j &= \sqrt{2P}\sin\left[\theta_{cj} + \Delta\, d_j\, Sqr(\theta_{scj})\right] + n_j \\ &= \sqrt{2P_c}\sin\theta_{cj} + \sqrt{2P_d}\, d_j\, Sqr(\theta_{scj})\cos\theta_{cj} + n_j \end{aligned} \tag{C-1}$$

where s_j is the sample at time t_j. The carrier and data powers, denoted P_c and P_d, are given by $P\cos^2\Delta$ and $P\sin^2\Delta$, respectively, and P is the total received signal power, Δ is the modulation index, $\theta_{cj} = \omega_c t_j + \theta_c$ is the total carrier phase at sample time t_j, n_j is the sampled form of an additive bandlimited white Gaussian noise process, d_j is the sampled form of the NRZ or Manchester data with a symbol time of T_s, and $Sqr()$ designates the sin square-wave subcarrier with total phase $\theta_{scj} = \omega_{sc} t_j + \theta_{sc}$. Here the first component, the residual carrier, typically is tracked by a phase-locked loop, and the second component, the suppressed carrier, can be tracked by a Costas loop. The narrowband noise n_j can be written as

$$n_j = \sqrt{2}n_{cj}\cos\theta_{cj} - \sqrt{2}n_{sj}\sin\theta_{cj} \tag{C-2}$$

where n_{cj} and n_{sj} are sampled versions of the statistically independent, stationary, bandlimited white Gaussian noise processes with one-sided spectral density level N_0 (W/Hz) and one-sided bandwidth B (Hz) that is large compared to $1/T_s$. The demodulation process requires carrier, subcarrier, and symbol synchronization. The following sections treat each of these subprocesses in order.

C.2 Carrier Demodulation

The reference signal for demodulating the carrier consists of an in-phase component (I) as well as a quadrature-phase component (Q). The Q-component is used to provide a tracking signal, while the I-component is used to obtain the actual data signal of interest. Thus, for the rest of this discussion, we will deal only with the I-component. The I-component reference signal is

$$r_{cj} = \sqrt{2}\cos\left(\hat{\theta}_{cj}\right) \tag{C-3}$$

where $\hat{\theta}_{cj}$ is the carrier-reference phase. Multiplying Eq. (C-1) by this reference signal yields

$$s'_j = \sqrt{P_c}\sin\phi_{cj} + \sqrt{P_d}\,d_j Sqr(\theta_{scj})C_{cj} + n'_{jj} \tag{C-4}$$

where the first term is due to any residual carrier and drops out as the carrier loop achieves lock. The second term is the data modulated onto the subcarrier, while the third term is the noise. The residual-carrier phase is $\phi_{cj} = \theta_{cj} - \hat{\theta}_{cj}$ and $C_{cj} = \cos\phi_{cj}$.

C.3 Subcarrier Demodulation

The reference signal for subcarrier demodulation, like the carrier, also has in-phase and quadrature-phase components:

$$r_{scj} = \begin{cases} Sqr\left(\hat{\theta}_{scj}\right) \\ Cqr\left(\hat{\theta}_{scj}\right) \end{cases} \tag{C-5}$$

where $\hat{\theta}_{scj}$ is the subcarrier-reference phase and $Cqr()$ designates the cosine subcarrier square wave. Multiplying Eq. (C-4) by this reference signal yields a result containing the two terms:

$$Sqr(\theta_{scj})xSqr\left(\hat{\theta}_{scj}\right)\underline{\Delta}C_{scj} = \left(1 - \frac{2}{\pi}\left|\phi_{scj}\right|\right)$$
$$Sqr(\theta_{scj})xCqr\left(\hat{\theta}_{scj}\right)\underline{\Delta}S_{scj} = \frac{2}{\pi}\phi_{scj} \tag{C-6}$$

where $\phi_{scj} = \theta_{scj} - \hat{\theta}_{scj}$, $\left|\phi_{scj}\right| < \left(\pi W_{sc}/2\right)$, and W_{sc} is the window in the subcarrier loop. The second term of Eq. (C-6) is the quadrature component, and it drops out as the subcarrier loop achieves lock. The in-phase component of the signal then becomes

$$s''_j = \sqrt{P_d}\, d_j\, C_{cj} C_{scj} + n''_j \tag{C-7}$$

C.4 Symbol Demodulation

The symbol modulation, d_j, is characterized by

$$d_j = \sum_{k=-\infty}^{\infty} d_k\, p(t_j - T_s) \tag{C-8}$$

where $d_k = \pm 1$ are the binary data , and $p(t)$ is the square-wave function that has a value of 1 for $0 \le t_j < T_s$ and a zero value elsewhere. The final output of the receiver, v_k, is then achieved by summing samples over the length of one symbol time, T_s , to get

$$v_k = \sum_{j=k+\lambda_{syk}}^{k+T_s+\lambda_{syk}} s''_j \tag{C-9}$$

where $\lambda_{syk} = (\theta_{syk} - \hat{\theta}_{sck}) / 2\pi = \phi_{syk} / 2\pi$ is the delay offset due to the symbol phase error. The symbol portion of the summation collapses to

$$\sum_{j=k+\lambda_{syk}}^{k+T_s+\lambda_{syk}} d_j\, p(t_j - T_s) = \left(1 - \left|\lambda_{syk}\right|\right) d_k = \left(1 - \frac{1}{2\pi}\left|\phi_{syk}\right|\right) d_k \tag{C-10}$$

Setting $C_{syk} = \left(1 - [1/2\pi] \mid \phi_{syk} \mid\right)$, the final receiver output reduces to

$$v_k = \sqrt{P_d}\, C_{ck} C_{sck} C_{syk} d_k + n_k \tag{C-11}$$

In practice, the time variation represented by k in the three reduction functions, C_{ck}, C_{sck}, and C_{syk}, is small.