Appendix B
Array Availability

Consider an array of n elements of equal G/T, where $n-m$ is required for successful operation, as discussed by Barlow et al. [1] and Jamnejad et al. [2]. The availability for each element is assumed to be equal to, but independent of, those for the other elements. No correlation is assumed between the failure rate or timing of different elements. Then the probability that at least $n-m$ elements are operating successfully at any given time can be calculated as follows. The probability that all the elements are operating successfully is

$$P_0 = p^n \tag{B-1}$$

and the probability that $n-1$ elements are operating successfully is equal to

$$P_1 = n(1-p)p^{n-1} \tag{B-2}$$

since this is the sum of n conditional probabilities for the case when one element is not functioning but the rest are. The probability that $n-2$ elements are operating successfully is then given by

$$p_2 = \left[\frac{n(n-1)}{2}\right](1-p)^2 p^{n-2} \tag{B-3}$$

This can be repeated until the case when only $n-m$ elements are operating, for which case we have

$$P_{n-m} = C(n,m)\,(1-p)^m\, p^{n-m} \tag{B-4}$$

in which

$$C(n,m) = \frac{n!}{[(n-m)!m!]} \tag{B-5}$$

is the number of combinations of m elements taken from a pool of n elements, and the "!" sign designates the factorial function.

The total probability of success for the array is then the sum of all the above cases,

$$P = \sum_{k=0}^{m} C(n,k)(1-p)^k p^{n-k} \tag{B-6}$$

which is also a form of the cumulative Bernoulli or binomial probability distribution function. Remember that we are comparing array elements having the same overall G/T. Assuming that T is more or less constant for the array, then the comparison is for array elements of equal G, or, equivalently, equal collecting aperture. Thus, for a total required collecting area of A (this can be either physical area or effective area, whichever is most convenient), the individual element area of an array of $n-m$ elements can be written as

$$A_{n-m} = \frac{a}{(n-m)} \tag{B-7}$$

Adding m marginal elements of area A_{n-m}, the incremental increase in the collecting area is mA_{n-m}, and the percentage of increase in the collecting area is given as

$$\frac{mA_{n-m}}{((n-m)A_{n-m})} = \frac{m}{(n-m)} \tag{B-8}$$

References

[1] R. E. Barlow and K. D. Heidtmann, "Computing *k-out-of-n* System Reliability," *IEEE Trans. on Reliability*, vol. R-33, no. 4, October 1984.

[2] V. Jamnejad, T. Cwik, and G. Resch, "Cost and Reliability Study for a Large Array of Small Reflector Antennas for JPL/NASA Deep Space Network (DSN)," *IEEE 1993 Aerospace Applications Conference Digest*, February 1993.