
LOW-ENERGY LUNAR TRAJECTORY DESIGN

**Jeffrey S. Parker and Rodney L. Anderson
Jet Propulsion Laboratory
Pasadena, California**

July 2013

DEEP SPACE COMMUNICATIONS AND NAVIGATION SERIES

Issued by the Deep Space Communications and Navigation Systems
Center of Excellence
Jet Propulsion Laboratory
California Institute of Technology

Joseph H. Yuen, Editor-in-Chief

Published Titles in this Series

Radiometric Tracking Techniques for Deep-Space Navigation
Catherine L. Thornton and James S. Border

*Formulation for Observed and Computed Values of Deep Space Network Data
Types for Navigation*
Theodore D. Moyer

*Bandwidth-Efficient Digital Modulation with Application to Deep-Space
Communication*
Marvin K. Simon

Large Antennas of the Deep Space Network
William A. Imbriale

Antenna Arraying Techniques in the Deep Space Network
David H. Rogstad, Alexander Mileant, and Timothy T. Pham

Radio Occultations Using Earth Satellites: A Wave Theory Treatment
William G. Melbourne

Deep Space Optical Communications
Hamid Hemmati, Editor

Spaceborne Antennas for Planetary Exploration
William A. Imbriale, Editor

Autonomous Software-Defined Radio Receivers for Deep Space Applications
Jon Hamkins and Marvin K. Simon, Editors

Low-Noise Systems in the Deep Space Network
Macgregor S. Reid, Editor

Coupled-Oscillator Based Active-Array Antennas
Ronald J. Pogorzelski and Apostolos Georgiadis

Low-Energy Lunar Trajectory Design
Jeffrey S. Parker and Rodney L. Anderson

LOW-ENERGY LUNAR TRAJECTORY DESIGN

**Jeffrey S. Parker and Rodney L. Anderson
Jet Propulsion Laboratory
Pasadena, California**

July 2013

Low-Energy Lunar Trajectory Design

July 2013



Jeffrey Parker:

*I dedicate the majority
of this book to my wife
Jen, my best friend and
greatest support
throughout the
development of this book
and always. I dedicate
the appendix to my son
Cameron, who showed up
right at the end.*

Rodney Anderson:

*I dedicate this book to
my wife Brooke for her
endless support and
encouragement.*

*We both thank our
families and friends for
their support throughout
the process.*

CONTENTS

Foreword	xi
Preface	xiii
Acknowledgments	xv
Authors	xxi
1 Introduction and Executive Summary	1
1.1 Purpose	1
1.2 Organization	1
1.3 Executive Summary	2
1.3.1 Direct, Conventional Transfers	5
1.3.2 Low-Energy Transfers	6
1.3.3 Summary: Low-Energy Transfers to Lunar Libration Orbits	7
1.3.4 Summary: Low-Energy Transfers to Low Lunar Orbits	8
1.3.5 Summary: Low-Energy Transfers to the Lunar Surface	10
1.4 Background	11
1.5 The Lunar Transfer Problem	12
1.6 Historical Missions	14
1.6.1 Missions Implementing Direct Lunar Transfers	15
1.6.2 Low-Energy Missions to the Sun–Earth Lagrange Points	15
1.6.3 Missions Implementing Low-Energy Lunar Transfers	20
1.7 Low-Energy Lunar Transfers	23
2 Methodology	27
2.1 Methodology Introduction	27
2.2 Physical Data	28
2.3 Time Systems	29
2.3.1 Dynamical Time, ET	29
2.3.2 International Atomic Time, TAI	29
2.3.3 Universal Time, UT	30
2.3.4 Coordinated Universal Time, UTC	30
2.3.5 Lunar Time	30
2.3.6 Local True Solar Time, LTST	31
2.3.7 Orbit Local Solar Time, OLST	31
2.4 Coordinate Frames	32
2.4.1 EME2000	32
2.4.2 EMO2000	33

2.4.3	Principal Axis Frame	33
2.4.4	IAU Frames	33
2.4.5	Synodic Frames	34
2.5	Models	35
2.5.1	CRTBP	36
2.5.2	Patched Three-Body Model	39
2.5.3	JPL Ephemeris	40
2.6	Low-Energy Mission Design	41
2.6.1	Dynamical Systems Theory	42
2.6.2	Solutions to the CRTBP	43
2.6.3	Poincaré Maps	49
2.6.4	The State Transition and Monodromy Matrices	50
2.6.5	Differential Correction	52
2.6.6	Constructing Periodic Orbits	67
2.6.7	The Continuation Method	74
2.6.8	Orbit Stability	77
2.6.9	Examples of Practical Three-Body Orbits	81
2.6.10	Invariant Manifolds	86
2.6.11	Orbit Transfers	95
2.6.12	Building Complex Orbit Chains	106
2.6.13	Discussion	113
2.7	Tools	114
2.7.1	Numerical Integrators	114
2.7.2	Optimizers	114
2.7.3	Software	115
3	Transfers to Lunar Libration Orbits	117
3.1	Executive Summary	117
3.2	Introduction	120
3.3	Direct Transfers Between Earth and Lunar Libration Orbits	122
3.3.1	Methodology	122
3.3.2	The Perigee-Point Scenario	125
3.3.3	The Open-Point Scenario	127
3.3.4	Surveying Direct Lunar Halo Orbit Transfers	130
3.3.5	Discussion of Results	152
3.3.6	Reducing the ΔV Cost	157
3.3.7	Conclusions	158
3.4	Low-Energy Transfers Between Earth and Lunar Libration Orbits	161
3.4.1	Modeling a Low-Energy Transfer using Dynamical Systems Theory	163
3.4.2	Energy Analysis of a Low-Energy Transfer	169
3.4.3	Constructing a Low-Energy Transfer in the Patched Three-Body Model	177
3.4.4	Constructing a Low-Energy Transfer in the Ephemeris Model of the Solar System	183

3.4.5	Families of Low-Energy Transfers	187
3.4.6	Monthly Variations in Low-Energy Transfers	190
3.4.7	Transfers to Other Three-Body Orbits	208
3.5	Three-Body Orbit Transfers	221
3.5.1	Transfers from an LL ₂ Halo Orbit to a Low Lunar Orbit	224
4	Transfers to Low Lunar Orbits	227
4.1	Executive Summary	227
4.2	Introduction	229
4.3	Direct Transfers Between Earth and Low Lunar Orbit	231
4.4	Low-Energy Transfers Between Earth and Low Lunar Orbit	233
4.4.1	Methodology	233
4.4.2	Example Survey	235
4.4.3	Arriving at a First-Quarter Moon	239
4.4.4	Arriving at a Third-Quarter Moon	246
4.4.5	Arriving at a Full Moon	250
4.4.6	Monthly Trends	253
4.4.7	Practical Considerations	257
4.4.8	Conclusions for Low-Energy Transfers Between Earth and Low Lunar Orbit	258
4.5	Transfers Between Lunar Libration Orbits and Low Lunar Orbits	258
4.6	Transfers Between Low Lunar Orbits and the Lunar Surface	258
5	Transfers to the Lunar Surface	263
5.1	Executive Summary	263
5.2	Introduction for Transfers to the Lunar Surface	265
5.3	Methodology	267
5.4	Analysis of Planar transfers between the Earth and the Lunar Surface	268
5.5	Low-Energy Spatial Transfers Between the Earth and the Lunar Surface	277
5.5.1	Trajectories Normal to the Surface	277
5.5.2	Trajectories Arriving at Various Angles to the Lunar Surface	287
5.6	Transfers Between Lunar Libration Orbits and the Lunar Surface	294
5.7	Transfers Between Low Lunar Orbits and the Lunar Surface	298
5.8	Conclusions Regarding Transfers to the Lunar Surface	298
6	Operations	299
6.1	Operations Executive Summary	299
6.2	Operations Introduction	300
6.3	Launch Sites	301
6.4	Launch Vehicles	301
6.5	Designing a Launch Period	304
6.5.1	Low-Energy Launch Periods	305
6.5.2	An Example Mission Scenario	307
6.5.3	Targeting Algorithm	311

6.5.4	Building a Launch Period	316
6.5.5	Reference Transfers	317
6.5.6	Statistical Costs of Desirable Missions to Low Lunar Orbit	317
6.5.7	Varying the LEO Inclination	325
6.5.8	Targeting a Realistic Mission to Other Destinations	328
6.5.9	Launch Period Design Summary	331
6.6	Navigation	332
6.6.1	Launch Targets	333
6.6.2	Station-Keeping	333
6.7	Spacecraft Systems Design	349
Appendix A: Locating the Lagrange Points		351
A.1	Introduction	351
A.2	Setting Up the System	351
A.3	Triangular Points	353
A.4	Collinear Points	354
A.4.1	Case 132: Identifying the L_1 point	355
A.4.2	Case 123: Identifying the L_2 point	355
A.4.3	Case 312: Identifying the L_3 point	356
A.5	Algorithms	357
A.5.1	Numerical Determination of L_1	357
A.5.2	Numerical Determination of L_2	358
A.5.3	Numerical Determination of L_3	358
References		359
Terms		377
Constants		382

FOREWORD

The Deep Space Communications and Navigation Systems Center of Excellence (DESCANSO) was established in 1998 by the National Aeronautics and Space Administration (NASA) at the California Institute of Technology's Jet Propulsion Laboratory (JPL). DESCANSO is chartered to harness and promote excellence and innovation to meet the communications and navigation needs of future deep-space exploration.

DESCANSO's vision is to achieve continuous communications and precise navigation—any time, anywhere. In support of that vision, DESCANSO aims to seek out and advocate new concepts, systems, and technologies; foster key technical talents; and sponsor seminars, workshops, and symposia to facilitate interaction and idea exchange.

The Deep Space Communications and Navigation Series, authored by scientists and engineers with many years of experience in their respective fields, lays a foundation for innovation by communicating state-of-the-art knowledge in key technologies. The series also captures fundamental principles and practices developed during decades of deep-space exploration at JPL. In addition, it celebrates successes and imparts lessons learned. Finally, the series will serve to guide a new generation of scientists and engineers.

Joseph H. Yuen, DESCANSO Leader

PREFACE

The purpose of this book is to provide high-level information to mission managers and detailed information to mission designers about low-energy transfers between the Earth and the Moon. This book surveys thousands of trajectories that one can use to transfer spacecraft between the Earth and various locations near the Moon, including lunar libration orbits, low lunar orbits, and the lunar surface. These surveys include conventional, direct transfers that require 3–6 days as well as more efficient, low-energy transfers that require more transfer time but which require less fuel. Low-energy transfers have been shown to be very useful in many circumstances and have recently been used to send satellites to the Moon, including the two *ARTEMIS* spacecraft and the two *GRAIL* spacecraft. This book illuminates the trade space of low-energy transfers and illustrates the techniques that may be used to build them.

ACKNOWLEDGMENTS

We would like to thank many people for their support writing this book, including people who have written or reviewed portions of the text, as well as people who have provided insight from years of experience flying spacecraft missions to the Moon and elsewhere. It is with sincere gratitude that we thank Ted Sweetser for his selfless efforts throughout this process, providing the opportunity for us to perform this work, and reviewing each section of this manuscript as it has come together. We would like to thank Al Cangahuala, Joe Guinn, Roby Wilson, and Amy Attiyah for their valuable feedback and thorough review of this work in each of its stages. We would also like to thank Tim McElrath for his feedback, insight, and excitement as we considered different aspects of this research.

We would like to give special thanks to several people who provided particular contributions to sections of the book. We thank Ralph Roncoli for his assistance with Sections 2.3 and 2.4, as well as his feedback throughout the book. Kate Davis assisted with Sections 2.6.3 and 2.6.11.3, most notably with the discussions of Poincaré sections. Roby Wilson provided particular assistance with Section 2.6.5 on the subject of the multiple shooting differential corrector. We would like to sincerely thank Andrew Peterson for his contribution to the development of Chapter 4. Finally, George Born and Martin Lo provided guidance for this research as it developed in

its early stages, leading to the authors' dissertations at the University of Colorado at Boulder.

Jeffrey Parker's Ph.D. dissertation (J. S. Parker, *Low-Energy Ballistic Lunar Transfers*, Ph.D. Thesis, University of Colorado, Boulder, 2007) provides the backbone to this manuscript and much of the dissertation has been repeated and amplified in this book. Much of the additional material that appears in this manuscript has been presented by the authors at conferences and published in journals. Such material has been reprinted here, with some significant alterations and additions. Finally, a number of additional journal articles and conference proceedings directly contributed to each chapter in the following list. In addition to their listing here, they are cited in text where the related material appears.

Chapter 2:

- J. S. Parker, K. E. Davis, and G. H. Born, "Chaining Periodic Three-Body Orbits in the Earth–Moon System," *ACTA Astronautica*, vol. 67, pp. 623–638, 2010.
- M. W. Lo, and J. S. Parker, "Chaining Simple Periodic Three-Body Orbits," *AAS/AIAA Astrodynamics Specialist Conference* (Lake Tahoe, California), Paper No. AAS 2005-380, August 7–11, 2005, vol. 123, *Advances in Astronautical Sciences* (B. G. Williams, L. A. D'Amario, K. C. Howell, and F. R. Hoots, editors), AAS/AIAA, Univelt Inc., San Diego, CA, 2006.
- R. B. Roncoli, *Lunar Constants and Models Document*, JPL D-32296 (internal document), Jet Propulsion Laboratory, California Institute of Technology, Pasadena, California, September 23, 2005.
- R. L. Anderson and J. S. Parker, "Survey of Ballistic Transfers to the Lunar Surface," *Journal of Guidance, Control, and Dynamics*, vol. 35, no. 4, pp. 1256–1267, July–August 2012.

Chapter 3:

- J. S. Parker, "Monthly Variations of Low-Energy Ballistic Transfers to Lunar Halo Orbits," *AIAA/AAS Astrodynamics Specialist Conference*, (Toronto, Ontario, Canada), Paper No. AIAA 2010-7963, August 2–5, 2010.
- J. S. Parker, "Targeting Low-Energy Ballistic Lunar Transfers," *AAS George H. Born Special Symposium*, (Boulder, Colorado), May 13–14, 2010, *American Astronautical Society*, 2010.
- J. S. Parker, "Targeting Low-Energy Ballistic Lunar Transfers," *Journal of Astronautical Sciences*, vol. 58, no. 3, pp. 311–334, July–September, 2011.

- J. S. Parker, “Low-Energy Ballistic Transfers to Lunar Halo Orbits,” *AAS/AIAA Astrodynamics Specialist Conference*, (Pittsburgh, Pennsylvania, Paper No. AAS 09-443, August 9–13, 2009, *Advances in Astronautical Sciences, Astrodynamics 2009* (A. V. Rao, A. Lovell, F. K. Chan, and L. A. Cangahuala, editors), vol. 135, pp. 2339–2358, 2010.
- J. S. Parker, and G. H. Born, “Modeling a Low-Energy Ballistic Lunar Transfer Using Dynamical Systems Theory,” *AIAA Journal of Spacecraft and Rockets*, vol. 45, no. 6, pp.1269–1281, November–December 2008.
- J. S. Parker and G. H. Born, “Direct Lunar Halo Orbit Transfers,” *Journal of the Astronautical Sciences*, vol. 56, issue 4, pp. 441–476, October–December 2008.
- J. S. Parker and G. H. Born, “Direct Lunar Halo Orbit Transfers,” *AAS/AIAA Spaceflight Mechanics Conference* (Sedona, Arizona, January 28–February 1, 2007), Paper No. AAS 07-229, *Advances in Astronautical Science*, vol. 127, pp. 1923–1945, 2007.
- J. S. Parker, “Families of Low-Energy Lunar Halo Transfers,” *AAS/AIAA Spaceflight Dynamics Conference*, (Tampa, Florida, January 22–26, 2006) Paper No. AAS 06-132, (S. R. Vadali, L. A. Cangahuala, J. P. W. Schumacher, and J. J. Guzman, editors), vol. 124 of *Advances in Astronautical Sciences*, San Diego, CA, AAS/AIAA, Univelt Inc., 2006.
- J. S. Parker and M. W. Lo, “Shoot the Moon 3D,” Paper AAS 05-383, *AAS/AIAA Astrodynamics Conference* held August 7–10, 2005, South Lake Tahoe, California, (originally published in) AAS publication, *Astrodynamics 2005* (edited by B. G. Williams, L. A. D’Amario, K. C. Howell, and F. R. Hoots) American Astronautical Society (AAS) *Advances in the Astronautical Sciences*, vol. 123, pp. 2067–2086, 2006, American Astronautical Society Publications Office, San Diego, California (Web Site: <http://www.univelt.com>), pp. 2067–2086.

Chapter 4:

- J. S. Parker and R. L. Anderson, “Targeting Low-Energy Transfers to Low Lunar Orbit,” *Astrodynamics: Proceedings of the 2011 AAS/AIAA Astrodynamics Specialist Conference*, (Girdwood, Alaska, July 31–August 4), Paper AAS 11-459, edited by H. Schaub, B. C. Gunter, R. P. Russell, and W. T. Cerven, Vol. 142, *Advances in the Astronautical Sciences*, American Astronautical Society, Univelt Inc., San Diego, California, pp. 847–866, 2012.
- J. S. Parker, R. L. Anderson, and A. Peterson, “A Survey of Ballistic Transfers to Low Lunar Orbit,” *21st AAS/AIAA Space Flight Mechanics Meeting*, (February 13–17, 2011, New Orleans, Louisiana), Paper AAS 11-277, Vol. 140, *Advances in the Astronautical Sciences* (edited by M. K. Jah, Y. Guo, A. L. Bowes, and

P. C. Lai), American Astronautical Society, Univelt Inc., San Diego, California, pp. 2461–2480, 2011.

Chapter 5:

- R. L. Anderson, and J. S. Parker, “Survey of Ballistic Transfers to the Lunar Surface,” *Journal of Guidance, Control, and Dynamics*, vol. 35, no. 4, pp. 1256–1267, July–August 2012.
- R. L. Anderson and J. S. Parker, “Comparison of Low-Energy Lunar Transfer Trajectories to Invariant Manifolds,” *Celestial Mechanics and Dynamical Astronomy*, vol. 115, DOI 10.1007/s10569-012-9466-3, pp. 311–331, published online February 16, 2013.
- R. L. Anderson, and J. S. Parker, “Comparison of Low-Energy Lunar Transfer Trajectories to Invariant Manifolds,” *AAS/AIAA Astrodynamics Specialist Conference* (Girdwood, Alaska, July 31–August 4, 2011), Paper AAS 11-423, edited by H. Schaub, B. C. Gunter, R. P. Russell, and W. T. Cerven, Vol. 142, *Advances in the Astronautical Sciences*, American Astronautical Society, Univelt Inc., San Diego, California, pp. 333–352, 2012.
- R. L. Anderson, and J. S. Parker, “A Survey of Ballistic Transfers to the Lunar Surface,” *Proceedings of the 21st AAS/AIAA Space Flight Mechanics Meeting* (New Orleans, Louisiana, February 13–17, 2011), Paper AAS 11-278, edited by M. K. Jah, Y. Guo, A. L. Bowes, and P. C. Lai, Vol. 140, *Advances in the Astronautical Sciences*, vol. 140, American Astronautical Society, Univelt Inc., San Diego, California, pp. 2481–2500, 2011.

Chapter 6:

- J. S. Parker, “Targeting Low-Energy Ballistic Lunar Transfers,” *Journal of Astronautical Sciences*, vol. 58, no. 3, pp. 311–334, July–September, 2011.
- J. S. Parker and R. L. Anderson, “Targeting Low-Energy Transfers to Low Lunar Orbit,” *Astrodynamics 2011: Proceedings of the AAS/AIAA Astrodynamics Specialist Conference* (Girdwood, Alaska, July 31–August 4, 2011), Paper AAS 11-459, edited by H. Schaub, B. C. Gunter, R. P. Russell, and W. T. Cerven, Vol. 142, *Advances in the Astronautical Sciences*, American Astronautical Society, Univelt Inc., San Diego, California, pp. 847–866, 2012.
- J. S. Parker, “Targeting Low-Energy Ballistic Lunar Transfers,” AAS 09-443, *AAS George H. Born Special Symposium* (Boulder, Colorado, May 13–14), American Astronautical Society, 2010.

A large portion of the research in this book, and all of the compiling of related research documentation from other sources, were carried out at the Jet Propulsion

Laboratory, California Institute of Technology, under a contract with the National Aeronautics and Space Administration. This work has been supported through funding by the Multimission Ground System and Services Office (MGSS) in support of the development of the Advanced Multi-Mission Operations System (AMMOS).

Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise, does not constitute or imply its endorsement by the United States Government or the Jet Propulsion Laboratory, California Institute of Technology.

Jeffrey S. Parker & Rodney L. Anderson

AUTHORS

Jeffrey S. Parker received his B.A. in 2001 in physics and astronomy from Whitman College (Walla Walla, Washington) and his M.S. and Ph.D. in aerospace engineering sciences from the University of Colorado at Boulder in 2003 and 2007, respectively. Dr. Parker was a member of the technical staff at the Jet Propulsion Laboratory (JPL) from January 2008 to June 2012. While at JPL he supported spacecraft exploration as a mission design and navigation specialist. He worked both as a spacecraft mission designer and as a navigator on the GRAIL mission, which sent two spacecraft to the Moon via low-energy ballistic lunar transfers. He supported India's *Chandrayaan-1* mission to the Moon, also as a mission designer and spacecraft navigator. Dr. Parker led the mission design development for numerous design studies and mission proposals, including missions to the Moon, near-Earth objects, the nearby Lagrange points, and most of the planets in the Solar System. At present, Dr. Parker is an assistant professor of astrodynamics at the University of Colorado at Boulder, teaching graduate and undergraduate courses in many subjects related to space exploration. His research interests are focused on astrodynamics and the exploration of space, including the design of low-energy trajectories in the Solar System, the optimization of low-thrust trajectories in the Solar System, autonomous spacecraft operations, and use of these engineering tools to provide new ways to achieve scientific objectives.

Rodney L. Anderson received his B.S. in 1997 in aerospace engineering from North Carolina State University at Raleigh and his M.S. and Ph.D. in aerospace engineering sciences from the University of Colorado at Boulder in 2001 and 2005, respectively. Upon the completion of his Ph.D., he worked as a research associate at the University of Colorado at Boulder conducting a study for the U.S. Air Force that focused on understanding the effects of atmospheric density variations on orbit predictions. Dr. Anderson has been a member of the JPL technical staff since 2010 where he has participated in mission design and navigation for multiple missions and continues to work on the development of new methods for trajectory design. His research interests are concentrated on the application of dynamical systems theory to astrodynamics and mission design. Some specific applications that he has focused on are the design of lunar trajectories, tour and endgame design in the Jovian system using heteroclinic connections, missions to near-Earth asteroids, and low-energy trajectories in multi-body systems. He has worked closely with multiple universities and has taught at both the University of Colorado at Boulder and the University of Southern California with an emphasis on the intersection of dynamical systems theory with astrodynamics.