Appendix G  
Simple Approximations: Spacecraft  
Surface Charging Equations  

Whereas Appendix D addresses internal charging analyses, this section will focus on surface charging.  

The simple approximations discussed in this section are of a worst-case nature. If this analysis indicates differential potentials between non-circuit surface materials of less than 400 V, there should be no spacecraft discharge problems. If predicted potentials on materials exceed 400 V, the Nascap-2k code (Appendix C.3.3) is to be used.  

Although the physics behind the spacecraft charging process is quite complex, the formulation at geosynchronous orbit can be expressed in very simple terms if a Maxwell-Boltzmann distribution is assumed. The fundamental physical process for all spacecraft charging is that of current balance; at equilibrium, all currents sum to zero. The potential at which equilibrium is achieved is the potential difference between the spacecraft and the space plasma ground. In terms of the current [1], the basic equation expressing this current balance for a given surface in an equilibrium situation is:  

\[ I_E(V) - [I_{IA}(V) + I_{SE}(V) + I_{SI}(V) + I_{BSE}(V) + I_{PH}(V) + I_B(V)] = I_T \]  

\[(G - 1)\]  

where:  

\[ V \quad = \quad \text{spacecraft potential} \]
\(I_E\) = incident electron current on spacecraft surface
\(I_I\) = incident ion current on spacecraft surface
\(I_{SE}\) = secondary electron current due to \(I_E\)
\(I_{SI}\) = secondary electron current due to \(I_I\)
\(I_{BSE}\) = backscattered electrons due to \(I_E\)
\(I_{PH}\) = photoelectron current
\(I_B\) = active current sources such as charged particle beams or ion thrusters
\(I_T\) = total current to spacecraft (at equilibrium, \(I_T = 0\)).

For a spherical body and a Maxwell-Boltzmann distribution, the first-order current densities (the current divided by the area over which the current is collected) can be calculated \([1]\) using the following equations (appropriate for small conducting sphere at GEO):

**Electrons**

\[
J_E = J_{E0} \exp\left(\frac{qV}{kT_E}\right) \quad V < 0 \text{ repelled} \quad (G-2)
\]
\[
J_E = J_{E0} \left[1 + \left(\frac{qV}{kT_E}\right)\right] \quad V > 0 \text{ attracted} \quad (G-3)
\]

**Ions**

\[
J_I = J_{I0} \exp\left(-\frac{qV}{kT_I}\right) \quad V > 0 \text{ repelled} \quad (G-4)
\]
\[
J_I = J_{I0} \left[1 - \left(\frac{qV}{kT_I}\right)\right] \quad V < 0 \text{ attracted} \quad (G-5)
\]

where:

\[
J_{E0} = \left(\frac{qN_E}{2}\right)\left(\frac{2kT_E}{\pi m_E}\right)^{1/2} \quad (G-6)
\]
\[
J_{I0} = \left(\frac{qN_I}{2}\right)\left(\frac{2kT_I}{\pi m_I}\right)^{1/2} \quad (G-7)
\]

where:

\(N_E\) = density of electrons
\(N_I\) = density of ions
\[ m_E = \text{mass of electrons} \]
\[ m_I = \text{mass of ions} \]
\[ q = \text{magnitude of the electronic charge}. \]
\[ T_E = \text{temperature of electrons} \]
\[ T_I = \text{temperature of ions} \]

Given these expressions and parameterizing the secondary and backscatter emissions, equation G-1 can be reduced to an analytic expression in terms of the potential at a point. This model, called an analytic probe model, can be stated as follows:

\[
A_E J_{EO} [1 – SE(V, T_E, N_E) – BSE(V, T_E, N_E)] \exp(qV/kT_E) \\
- A_I J_{I0} [1 + SI(V, T_I, N_I)][1 – (qV/kT_I)] \\
- A_{PH} J_{PHO} f(X_m) = I_T = 0 \quad V < 0 \tag{G-8}
\]

where:
\[ A_E = \text{electron collection area} \]
\[ J_{EO} = \text{ambient electron current density} \]
\[ A_I = \text{ion collection area} \]
\[ J_{I0} = \text{ambient ion current density} \]
\[ A_{PH} = \text{photoelectron emission area} \]
\[ J_{PHO} = \text{saturation photoelectron flux} \]

\[ BSE, SE, SI = \text{parameterization functions for secondary emission related to backscatter, electrons, and ions} \]

\[ f(X_m) = \text{attenuated solar flux as a function of altitude } X_m \text{ of center of Sun above the surface of Earth as seen by spacecraft (percent)}. \]

This equation is appropriate for a small (<10 m), uniformly conducting spacecraft at geosynchronous orbit in the absence of magnetic field effects. To solve the equation, \( V \) is varied until \( I_T = 0 \). Typical values for aluminum of \( SI, SE, \) and \( BSE \) are 3, 0.4, and 0.2, respectively. For geosynchronous orbit, \( J_E/J_I \) is about 30 during a geomagnetic storm.
As discussed earlier in Eq. (2.3-6), when the spacecraft is in eclipse (and
ingnoring secondary and backscattered terms), a simple proportionality between
the satellite potential and the currents and temperature can be derived from
Eq. (G-8):

\[ V \sim \frac{-T_E}{q} \times \ln\left(\frac{J_E}{J_I}\right) \]  \hspace{1cm} (G-9)

where:

- \( T_E \) is in eV.

That is, to rough order in eclipse, the spacecraft potential is directly
proportional to the plasma temperature expressed in electron volts and the
natural log of the ratio of the electron and ion currents. Note, however, that
secondary currents play a crucial role in actual calculations, and \( T_E \) must be
greater than some critical value [2–5], usually of the order of 1000 eV, before
charging will occur because secondary electron production can exceed the
ambient current for low enough \( T_E \). Also, \( \ln(J_E/J_I) \) often varies much more
rapidly and by larger factors than \( T_E \) so that charging has been found often to be
more related to changes in \( \ln(J_E/J_I) \) than \( T_E \) [6].

References

Geophysics and Space Physics*, vol. 19, no. 4, pp. 577–616, November
1981.

A nice summary paper, with numerical examples and many illustrations.
This and Whipple (1981) [7] are two definitive papers on the subject,
each covering slightly different aspects.

Geosynchronous Plasma Environment,” *Planetary and Space Science*,


Altitudes: New Evidence for Existence of Critical Temperature,” *Journal of

Measured Geosynchronous Plasma Environment in Spacecraft Charging
Calculations,” *Journal of Geophysical Research*, vol. 113, no. A10204,


Nice summary paper. Emphasis is on total charging and not internal charging, but good for physics background. This and Garrett (1981) [1] are the two definitive papers on that subject, each covering slightly different aspects.