Appendix B
Bistatic Scattering Matrix of a Cylinder with Arbitrary Orientation

In this appendix we summarize the equations describing the bistatic scattering from a finite length dielectric cylinder with arbitrary orientation. To describe the scattering, we shall use a global coordinate system (see Fig. B-1) wherein the cylinder orientation is described using a unit vector aligned with the long axis of the cylinder, as follows:

$$\mathbf{c}\left(\theta_c, \phi_c\right) = \sin\theta_c \cos\phi_c \, \mathbf{x} + \sin\theta_c \sin\phi_c \, \mathbf{y} + \cos\theta_c \, \mathbf{z} \tag{B.1}$$

We shall assume that an electromagnetic wave is incident upon this cylinder in such a way that the propagation vector of the incident wave can be written as

$$\mathbf{k}_i = -\sin\theta_i \cos\phi_i \, \mathbf{x} - \sin\theta_i \sin\phi_i \, \mathbf{y} - \cos\theta_i \, \mathbf{z} \tag{B.2}$$

We are interested in the scattered field that is represented by the propagation vector

$$\mathbf{k}_s = -\sin\theta_s \cos\phi_s \, \mathbf{x} - \sin\theta_s \sin\phi_s \, \mathbf{y} - \cos\theta_s \, \mathbf{z} \, . \tag{B.3}$$

Using the backscatter alignment coordinate system, we define two triplets of local coordinates to describe the transverse components of the incident and scattered fields. These coordinates are defined as

$$\mathbf{v}_i = -\cos\theta_i \cos\phi_i \, \mathbf{x} - \cos\theta_i \sin\phi_i \, \mathbf{y} + \sin\theta_i \, \mathbf{z} \, , \tag{B.4}$$

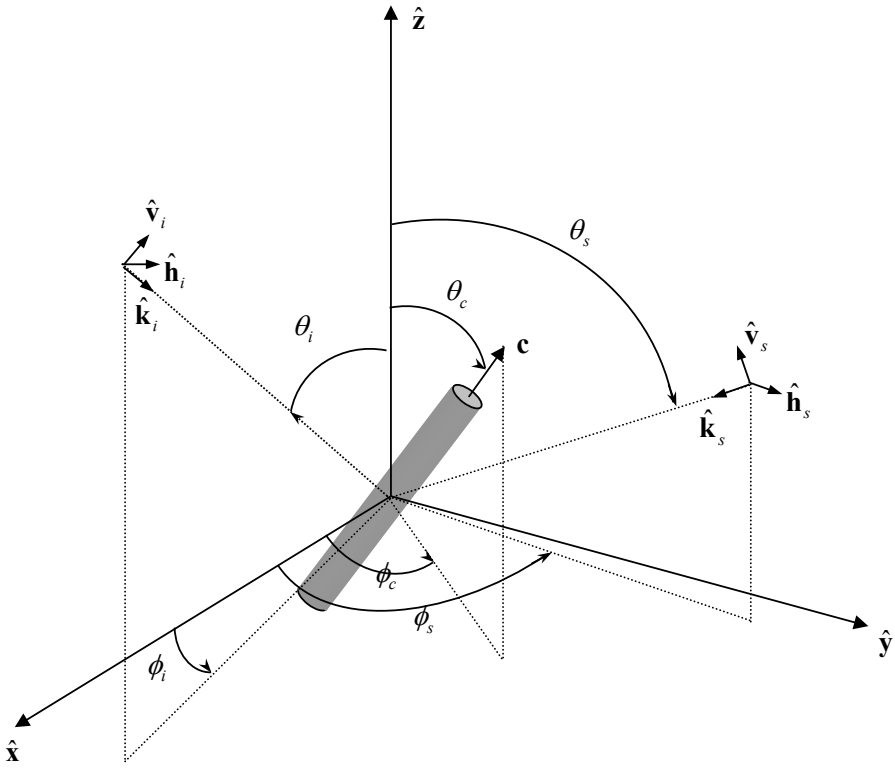

Fig. B-1. Global backscattering alignment coordinate system used in the calculations.

$$\mathbf{h}_i = \sin\phi_i\,\mathbf{x} - \cos\phi_i\,\mathbf{y}\,, \tag{B.5}$$

$$\mathbf{v}_s = -\cos\theta_s\cos\phi_s\,\mathbf{x} - \cos\theta_s\sin\phi_s\,\mathbf{y} + \sin\theta_s\,\mathbf{z}\,, \tag{B.6}$$

and

$$\mathbf{h}_s = \sin\phi_s\,\mathbf{x} - \cos\phi_s\,\mathbf{y}\,. \tag{B.7}$$

We shall assume that the incident wave is a plane electromagnetic wave. Because the cylinder is of finite size, the scattered wave is, in general, a spherical wave that propagates away from the cylinder. We shall assume that we know the expressions for the bistatic scattering matrix of a cylinder that is oriented vertically and denote this matrix by $\mathbf{S}(\theta_i,\phi_i,\theta_s,\phi_s)$. We shall use the same definition as proposed by van Zyl and Ulaby [1], which relates the incident and scattered waves as follows:

$$\mathbf{E}^{sc} = [\mathbf{S}]\mathbf{E}^{inc}\frac{e^{ikr}}{r}\,. \tag{B.8}$$

The term on the extreme right $\left(\frac{e^{ikr}}{kr}\right)$ explicitly shows the amplitude and phase of a spherical wave. Note that, by this definition, the scattering matrix is not dimensionless. Van de Hulst [2] defines the denominator of the spherical wave as *kr*; in Van de Hulst's definition the scattering matrix is dimensionless. In our case, the dimension of the scattering matrix elements is meters.

The elements of the bistatic scattering matrix for a vertically oriented cylinder are derived by Senior and Sarabandi [3] as

$$S_{hh}\left(\theta_i,\phi_i,\theta_s,\phi_s\right)=-\frac{1}{2}\frac{k_0 al}{\sin\theta_i}\frac{\sin\left(k_0 l\left(\cos\theta_i+\cos\theta_s\right)/2\right)}{k_0 l\left(\cos\theta_i+\cos\theta_s\right)/2}D^h, \tag{B.9}$$

$$S_{hv}\left(\theta_i,\phi_i,\theta_s,\phi_s\right)=-\frac{1}{2}\frac{k_0 al}{\sin\theta_i}\frac{\sin\left(k_0 l\left(\cos\theta_i+\cos\theta_s\right)/2\right)}{k_0 l\left(\cos\theta_i+\cos\theta_s\right)/2}i\bar{D}^h, \tag{B.10}$$

$$S_{vh}\left(\theta_i,\phi_i,\theta_s,\phi_s\right)=+\frac{1}{2}\frac{k_0 al}{\sin\theta_i}\frac{\sin\left(k_0 l\left(\cos\theta_i+\cos\theta_s\right)/2\right)}{k_0 l\left(\cos\theta_i+\cos\theta_s\right)/2}i\bar{D}^e, \tag{B.11}$$

and

$$S_{vv}\left(\theta_i,\phi_i,\theta_s,\phi_s\right)=-\frac{1}{2}\frac{k_0 al}{\sin\theta_i}\frac{\sin\left(k_0 l\left(\cos\theta_i+\cos\theta_s\right)/2\right)}{k_0 l\left(\cos\theta_i+\cos\theta_s\right)/2}D^e. \tag{B.12}$$

In these equations, a is the radius of the cylinder, $k_0=2\pi/\lambda$ is the wave number of the incident wave, and l is the length of the cylinder. Also,

$$\begin{aligned}D^e=\sum_{m=-\infty}^{+\infty}(-1)^m\Bigg\{&J_m'(x_0)J_m(y_0)-\frac{\sin\theta_i}{B}J_m(x_0)J_m'(y_0)\\&+C_m^{TM}\left[H_m^{(1)\prime}(x_0)J_m(y_0)-\frac{\sin\theta_i}{B}H_m^{(1)}(x_0)J_m'(y_0)\right]\\&+\frac{m\cos\theta_i}{x_0}\left(1-\frac{\mathbf{k}_s\cdot\mathbf{c}(0,0)}{\cos\theta_i}\frac{x_0\sin\theta_i}{y_0 B}\right)\bar{C}_m H_m^{(1)}(x_0)J_m(y_0)\Bigg\}e^{im\tilde{\phi}}\end{aligned}, \tag{B.13}$$

$$D^h = \sum_{m=-\infty}^{+\infty} (-1)^m \left\{ J'_m(x_0) J_m(y_0) - \frac{\sin\theta_i}{B} J_m(x_0) J'_m(y_0) \right.$$
$$+ C_m^{TE} \left[H_m^{(1)\prime}(x_0) J_m(y_0) - \frac{\sin\theta_i}{B} H_m^{(1)}(x_0) J'_m(y_0) \right] \quad , \tag{B.14}$$
$$\left. + \frac{m\cos\theta_i}{x_0} \left(1 - \frac{\mathbf{k}_s \cdot \mathbf{c}(0,0)}{\cos\theta_i} \frac{x_0 \sin\theta_i}{y_0 B} \right) \bar{C}_m H_m^{(1)}(x_0) J_m(y_0) \right\} e^{im\tilde{\phi}}$$

$$\bar{D}^e = \sum_{m=-\infty}^{+\infty} (-1)^m e^{im\tilde{\phi}} \left\{ \bar{C}_m \left[H_m^{(1)\prime}(x_0) J_m(y_0) - \frac{\sin\theta_i}{B} H_m^{(1)}(x_0) J'_m(y_0) \right] \right. \quad , \tag{B.15}$$
$$\left. + \frac{m\cos\theta_i}{x_0} \left(1 - \frac{\mathbf{k}_s \cdot \mathbf{c}(0,0)}{\cos\theta_i} \frac{x_0 \sin\theta_i}{y_0 B} \right) \left[J_m(x_0) + C_m^{TE} H_m^{(1)}(x_0) \right] J_m(x_0) \right\}$$

$$\bar{D}^h = \sum_{m=-\infty}^{+\infty} (-1)^m e^{im\tilde{\phi}} \left\{ \bar{C}_m \left[H_m^{(1)\prime}(x_0) J_m(y_0) - \frac{\sin\theta_i}{B} H_m^{(1)}(x_0) J'_m(y_0) \right] \right. \quad , \tag{B.16}$$
$$\left. + \frac{m\cos\theta_i}{x_0} \left(1 - \frac{\mathbf{k}_s \cdot \mathbf{c}(0,0)}{\cos\theta_i} \frac{x_0 \sin\theta_i}{y_0 B} \right) \left[J_m(x_0) + C_m^{TM} H_m^{(1)}(x_0) \right] J_m(x_0) \right\}$$

$$B = \frac{1}{2} \sqrt{\left(\sin\theta_i + \sin\theta_s \cos(\phi_s - \phi_i) \right)^2 + \sin^2\theta_s \cos^2(\phi_s - \phi_i)^2} \ , \tag{B.17}$$

$$\cos\tilde{\phi} = \frac{1}{2B} \left(\sin\theta_s \cos(\phi_s - \phi_i) + \sin\theta_i \right); \quad \sin\tilde{\phi} = \frac{1}{2B} \sin\theta_s \sin(\phi_s - \phi_i) \ , \tag{B.18}$$

$$y_0 = k_0 a B \ , \tag{B.19}$$

$$x_0 = k_0 a \sin\theta_i \ , \tag{B.20}$$

and

$$C_m^{TM} = -\frac{V_m P_m - q_m^2 J_m(x_0) H_m^{(1)}(x_0) J_m^2(x_1)}{P_m N_m - \left[q_m H_m^{(1)}(x_0) J_m(x_1) \right]^2} \ , \tag{B.21}$$

$$C_m^{TE} = -\frac{M_m N_m - q_m^2 J_m(x_0) H_m^{(1)}(x_0) J_m^2(x_1)}{P_m N_m - \left[q_m H_m^{(1)}(x_0) J_m(x_1) \right]^2} \ , \tag{B.22}$$

$$\bar{C}_m = i \frac{2}{\pi x_0 \sin\theta_i} \frac{q_m J_m^2(x_1)}{P_m N_m - \left[q_m H_m^{(1)}(x_0) J_m(x_1) \right]^2} \ , \tag{B.23}$$

with

$$x_1 = k_0 a \sqrt{\varepsilon - \cos^2 \theta_i} \ , \tag{B.24}$$

$$q_m = m k_0 a \cos\theta_i \left(\frac{1}{x_1^2} - \frac{1}{x_0^2} \right), \tag{B.25}$$

$$V_m = k_0 a \left\{ \frac{\varepsilon}{x_1} J_m(x_0) J'_m(x_1) - \frac{1}{x_0} J'_m(x_0) J_m(x_1) \right\}, \tag{B.26}$$

$$P_m = k_0 a \left\{ \frac{1}{x_1} H_m^{(1)}(x_0) J'_m(x_1) - \frac{1}{x_0} H_m^{(1)\prime}(x_0) J_m(x_1) \right\}, \tag{B.27}$$

$$N_m = k_0 a \left\{ \frac{\varepsilon}{x_1} H_m^{(1)}(x_0) J'_m(x_1) - \frac{1}{x_0} H_m^{(1)\prime}(x_0) J_m(x_1) \right\}, \tag{B.28}$$

and

$$M_m = k_0 a \left\{ \frac{1}{x_1} J_m(x_0) J'_m(x_1) - \frac{1}{x_0} J'_m(x_0) J_m(x_1) \right\}. \tag{B.29}$$

The cylinder complex dielectric constant is ε. Note the dimension of each element of the scattering matrix.

Equations (B.9) – (B.29) apply to the case of a vertically oriented cylinder. Returning to the case of a cylinder with arbitrary orientation, we will now define two local coordinate systems for the incident and scattered waves such that we can use these expressions to characterize the scattering in those two coordinate systems. We shall denote these coordinate systems by primed vectors. Starting with the incident electric field, we note that we can write this field as

$$\mathbf{E}^{inc} = E_h^{inc} \mathbf{h}_i + E_v^{inc} \mathbf{v}_i = E_{h'}^{inc} \mathbf{h}'_i + E_{v'}^{inc} \mathbf{v}'_i, \tag{B.30}$$

from which it is easily shown that

$$\begin{pmatrix} E_{h'} \\ E_{v'} \end{pmatrix}^{inc} = \begin{pmatrix} \mathbf{h}_i \cdot \mathbf{h}'_i & \mathbf{v}_i \cdot \mathbf{h}'_i \\ \mathbf{h}_i \cdot \mathbf{v}'_i & \mathbf{v}_i \cdot \mathbf{v}'_i \end{pmatrix} \begin{pmatrix} E_h \\ E_v \end{pmatrix}^{inc} . \tag{B.31}$$

The bistatic scattering matrix links the incident and scattered waves in the local coordinate systems aligned with the cylinder axis, as follows:

$$\begin{pmatrix} E_{h'} \\ E_{v'} \end{pmatrix}^{sc} = \mathbf{S}\left(\theta_{ic}, \phi_{ic}, \theta_{sc}, \phi_{sc}\right) \begin{pmatrix} E_{h'} \\ E_{v'} \end{pmatrix}^{inc}, \tag{B.32}$$

where the subscripts ic and sc indicate that the angles are relative to the cylinder orientation, rather than the z-axis, as was the case in Eqs. (B.9) – (B.29).

The scattered wave can also be written as

$$\mathbf{E}^{sc} = E_s^{sc}\mathbf{h}_s + E_s^{sc}\mathbf{v}_s = E_{h'}^{sc}\mathbf{h}'_s + E_{v'}^{sc}\mathbf{v}'_s, \tag{B.33}$$

from which we can show that

$$\begin{pmatrix} E_h \\ E_v \end{pmatrix}^{sc} = \begin{pmatrix} \mathbf{h}_s \cdot \mathbf{h}'_s & \mathbf{h}_s \cdot \mathbf{v}'_s \\ \mathbf{v}_s \cdot \mathbf{h}'_s & \mathbf{v}_s \cdot \mathbf{v}'_s \end{pmatrix} \begin{pmatrix} E_{h'} \\ E_{v'} \end{pmatrix}^{sc}. \tag{B.34}$$

Combining Eqs. (B.31), (B.32), and (B.34), we find the bistatic scattering matrix of the cylinder as

$$\mathbf{S}\left(\theta_i, \phi_i, \theta_s, \phi_s, \theta_c, \phi_c\right) = \begin{pmatrix} \mathbf{h}_s \cdot \mathbf{h}'_s & \mathbf{h}_s \cdot \mathbf{v}'_s \\ \mathbf{v}_s \cdot \mathbf{h}'_s & \mathbf{v}_s \cdot \mathbf{v}'_s \end{pmatrix} \mathbf{S}\left(\theta_{ic}, \phi_i, \theta_{sc}, \phi_s\right) \begin{pmatrix} \mathbf{h}_i \cdot \mathbf{h}'_i & \mathbf{v}_i \cdot \mathbf{h}'_i \\ \mathbf{h}_i \cdot \mathbf{v}'_i & \mathbf{v}_i \cdot \mathbf{v}'_i \end{pmatrix}. \tag{B.35}$$

The local coordinate systems are defined as

$$\mathbf{h}'_i = \frac{\mathbf{k}_i \times \mathbf{c}\left(\theta_c, \phi_c\right)}{\left|\mathbf{k}_i \times \mathbf{c}\left(\theta_c, \phi_c\right)\right|} \tag{B.36}$$

and

$$\mathbf{v}'_i = \mathbf{h}'_i \times \mathbf{k}_i = \frac{1}{\left|\mathbf{k}_i \times \mathbf{c}\left(\theta_c, \phi_c\right)\right|} \left\{\mathbf{c}\left(\theta_c, \phi_c\right) - \mathbf{k}_i\left(\mathbf{k}_i \cdot \mathbf{c}\left(\theta_c, \phi_c\right)\right)\right\}. \tag{B.37}$$

The scattered wave coordinate axes are defined in the same way. We note that we can write

$$\left|\mathbf{k}_i \times \mathbf{c}\left(\theta_c, \phi_c\right)\right| = \sqrt{1 - \left(\mathbf{k}_i \cdot \mathbf{c}\left(\theta_c, \phi_c\right)\right)^2}. \tag{B.38}$$

Also, note that the unprimed coordinate systems can be written like Eq. (B.36) and Eq. (B.37) with $\mathbf{c}\left(\theta_c, \phi_c\right)$ replaced by $\mathbf{z}$. It then follows that

$$\mathbf{h}_i \cdot \mathbf{h}_i' = \frac{\mathbf{c}(\theta_c,\phi_c)\cdot\mathbf{z} - (\mathbf{k}_i\cdot\mathbf{z})(\mathbf{k}_i\cdot\mathbf{c}(\theta_c,\phi_c))}{\sqrt{1-(\mathbf{k}_i\cdot\mathbf{z})^2}\sqrt{1-(\mathbf{k}_i\cdot\mathbf{c}(\theta_c,\phi_c))^2}} \quad , \tag{B.39}$$

$$\mathbf{v}_i \cdot \mathbf{v}_i' = \mathbf{h}_i \cdot \mathbf{h}_i' , \tag{B.40}$$

and

$$\mathbf{h}_i \cdot \mathbf{v}_i' = -\mathbf{v}_i \cdot \mathbf{h}_i' = -\frac{\mathbf{k}_i\cdot(\mathbf{c}(\theta_c,\phi_c)\times\mathbf{z})}{\sqrt{1-(\mathbf{k}_i\cdot\mathbf{z})^2}\sqrt{1-(\mathbf{k}_i\cdot\mathbf{c}(\theta_c,\phi_c))^2}} \quad . \tag{B.41}$$

It can be shown that

$$\begin{aligned}&\mathbf{c}(\theta_c,\phi_c)\cdot\mathbf{z} - (\mathbf{k}_i\cdot\mathbf{z})(\mathbf{k}_i\cdot\mathbf{c}(\theta_c,\phi_c))\\ &= \sin\theta_i\left[\cos\theta_c\sin\theta_i - \sin\theta_c\cos\theta_i\cos(\phi_c-\phi_i)\right]\end{aligned} \tag{B.42}$$

and

$$\mathbf{k}_i\cdot(\mathbf{c}(\theta_c,\phi_c)\times\mathbf{z}) = -\sin\theta_i\sin\theta_c\sin(\phi_c-\phi_i) . \tag{B.43}$$

Using Eq. (B.43) and Eq. (B.42) in Eq. (B.41) and Eq. (B.39), respectively, we find that

$$\mathbf{h}_i \cdot \mathbf{h}_i' = \mathbf{v}_i \cdot \mathbf{v}_i' = \frac{1}{\sin\theta_{ic}}\left\{\cos\theta_c\sin\theta_i - \sin\theta_c\cos\theta_i\cos(\phi_c-\phi_i)\right\} , \tag{B.44}$$

and

$$\mathbf{h}_i \cdot \mathbf{v}_i' = -\mathbf{v}_i \cdot \mathbf{h}_i' = -\frac{\sin\theta_c\sin(\phi_c-\phi_i)}{\sin\theta_{ic}} . \tag{B.45}$$

For the scattered wave, we find that

$$\mathbf{h}_s \cdot \mathbf{h}_s' = \mathbf{v}_s \cdot \mathbf{v}_s' = \frac{\mathbf{c}(\theta_c,\phi_c)\cdot\mathbf{z} - (\mathbf{k}_s\cdot\mathbf{z})(\mathbf{k}_s\cdot\mathbf{c}(\theta_c,\phi_c))}{\sqrt{1-(\mathbf{k}_s\cdot\mathbf{z})^2}\sqrt{1-(\mathbf{k}_s\cdot\mathbf{c}(\theta_c,\phi_c))^2}} \tag{B.46}$$

and

$$\mathbf{h}_s \cdot \mathbf{v}_s' = -\mathbf{v}_s \cdot \mathbf{h}_s' = -\frac{\mathbf{k}_s\cdot(\mathbf{c}(\theta_c,\phi_c)\times\mathbf{z})}{\sqrt{1-(\mathbf{k}_s\cdot\mathbf{z})^2}\sqrt{1-(\mathbf{k}_s\cdot\mathbf{c}(\theta_c,\phi_c))^2}} \quad , \tag{B.47}$$

which can be written as

$$\mathbf{h}_s \cdot \mathbf{h}'_s = \mathbf{v}_s \cdot \mathbf{v}'_s = \frac{1}{\sin\theta_{sc}} \left\{\cos\theta_c \sin\theta_s - \sin\theta_c \cos\theta_s \cos\left(\phi_c - \phi_s\right)\right\} \tag{B.48}$$

and

$$\mathbf{h}_s \cdot \mathbf{v}'_s = -\mathbf{v}_s \cdot \mathbf{h}'_s = -\frac{\sin\theta_c \sin\left(\phi_c - \phi_s\right)}{\sin\theta_{sc}}. \tag{B.49}$$

The angles θ_{ic} and θ_{sc} are defined by

$$\cos\theta_{ic} = -\left(\mathbf{k}_i \cdot \mathbf{c}\right) = \cos\theta_c \cos\theta_i + \sin\theta_c \sin\theta_i \cos\left(\phi_c - \phi_i\right) \tag{B.50}$$

and

$$\cos\theta_{sc} = -\left(\mathbf{k}_s \cdot \mathbf{c}\right) = \cos\theta_c \cos\theta_s + \sin\theta_c \sin\theta_s \cos\left(\phi_c - \phi_s\right). \tag{B.51}$$

It is also useful to look at simpler expressions reported in the literature. Barrick [4] used the expressions for the fields scattered by an infinitely long cylinder and then accounted for the finite length l of the cylinder by multiplying by a factor

$$l\sqrt{\frac{k_0}{\pi}} \sin\theta_{sc} \frac{\sin\left[k_0 l\left(\cos\theta_{ic} + \cos\theta_{sc}\right)/2\right]}{k_0 l\left(\cos\theta_{ic} + \cos\theta_{sc}\right)/2}. \tag{B.52}$$

Under this assumption, and evaluating Eqs. (B.9) – (B.12) on the cone $B = \sin\theta_i$, we find

$$S_{hh}\left(\theta_i, \phi_i, \theta_s, \phi_s\right) = -\frac{il\sin\theta_s}{\pi\sin\theta_i} \frac{\sin V}{V} \sum_{m=-\infty}^{+\infty} \left\{\left(-1\right)^m C_m^{TM} e^{im\left(\phi_s - \phi_i\right)}\right\}, \tag{B.53}$$

$$S_{hv}\left(\theta_i, \phi_i, \theta_s, \phi_s\right) = -\frac{il\sin\theta_s}{\pi\sin\theta_i} \frac{\sin V}{V} \sum_{m=-\infty}^{+\infty} \left\{\left(-1\right)^m \overline{C}_m e^{im\left(\phi_s - \phi_i\right)}\right\}, \tag{B.54}$$

$$S_{vh}\left(\theta_i, \phi_i, \theta_s, \phi_s\right) = +\frac{il\sin\theta_s}{\pi\sin\theta_i} \frac{\sin V}{V} \sum_{m=-\infty}^{+\infty} \left\{\left(-1\right)^m \overline{C}_m e^{im\left(\phi_s - \phi_i\right)}\right\}, \tag{B.55}$$

and

$$S_{vv}\left(\theta_i,\phi_i,\theta_s,\phi_s\right)=-\frac{il\sin\theta_s}{\pi\sin\theta_i}\frac{\sin V}{V}\sum_{m=-\infty}^{+\infty}\left\{\left(-1\right)^m C_m^{TE}e^{im\left(\phi_s-\phi_i\right)}\right\}, \tag{B.56}$$

where

$$V=\frac{1}{2}k_0 l\left(\cos\theta_i+\cos\theta_s\right). \tag{B.57}$$

Note that the series coefficients given in Eqs. (B.21) – (B.29) have the following symmetry relations

$$C_m^{TM}=C_{-m}^{TM};\quad C_m^{TE}=C_{-m}^{TE};\quad \overline{C}_m=-\overline{C}_{-m};\quad \overline{C}_0=0. \tag{B.58}$$

This means that the far-zone scattered field in the plane of incidence is not depolarized. This, however, is not the case of other azimuth angles. Also, in the forward scattering direction, required for the calculation of the extinction coefficient, these expressions are

$$S_{hh}\left(\theta_i,\phi_i,\pi-\theta_i,\phi_i+\pi\right)=-\frac{il\sin\theta_s}{\pi\sin\theta_i}\frac{\sin V}{V}\sum_{m=0}^{+\infty}C_m^{TM}, \tag{B.59}$$

$$S_{hv}\left(\theta_i,\phi_i,\pi-\theta_i,\phi_i+\pi\right)=S_{vh}\left(\theta_i,\phi_i,\pi-\theta_s,\phi_i+\pi\right)=0, \tag{B.60}$$

and

$$S_{vv}\left(\theta_i,\phi_i,\pi-\theta_i,\phi_i+\pi\right)=-\frac{il\sin\theta_s}{\pi\sin\theta_i}\frac{\sin V}{V}\sum_{m=0}^{+\infty}C_m^{TE}. \tag{B.61}$$

Senior and Sarabandi [3] built on the work of Barrick [4] by integrating the current distribution over a cylinder of finite length. In contrast, Barrick [4] added a $\sin x/x$ term to the scattered field of an infinitely long cylinder to account for the finite length. As such, Senior and Sarabandi's equations are possibly more accurate than those of Barrick, but require significantly more calculations.

We shall use these expressions to calculate the composite scattering from a layer of vegetation. If we consider such a layer, there are three basic calculations for the layer as a whole that we need to perform. These include the backscatter from the layer, the bistatic forward scattering that would interact with the ground surface, and the attenuation through the layer. The latter is needed to calculate the attenuated backscatter from the underlying soil surface. The expressions listed so far can be used directly to calculate the first two

components. To calculate the attenuation through the layer, we shall make use of the optical theorem that states that the extinction cross section of a single particle is related to the forward scattering field through

$$\sigma_p^e = \frac{2\pi}{k_0} \mathrm{Im}\left[S_{pp}\left(\theta_{ic}, \phi_i, \pi - \theta_{ic}, \phi_i + \pi, \theta_c, \phi_c\right)\right], \tag{B.62}$$

where p denotes the polarization of the wave. The dimension of this is meters squared, since the scattering matrix has dimension meters. The total extinction coefficient of a medium containing a random distribution of N cylinders per unit volume is obtained by performing an ensemble average over the particles, as follows:

$$\kappa_p^e = N\left\langle \sigma_p^e \right\rangle . \tag{B.63}$$

The dimension of this quantity is m^{-1}. Using this definition, the strength of the incident wave after propagating through a layer of thickness d at an angle θ_i with respect to the vertical direction, is given by

$$\begin{pmatrix} E_h \\ E_v \end{pmatrix}^{tr} = \begin{pmatrix} e^{-\kappa_h^e d/\cos\theta_i} & 0 \\ 0 & e^{-\kappa_v^e d/\cos\theta_i} \end{pmatrix} \begin{pmatrix} E_h \\ E_v \end{pmatrix}^{inc} . \tag{B.64}$$

To find the total field propagating in this direction, we need to add the bistatic scattered field in this direction to Eq. (B.64).

With these expressions for the scattering from an arbitrarily oriented cylinder, one can define models to describe the scattering from vegetated terrain.

References

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[2] H. C. van de Hulst, *Light Scattering by Small Particles*, Dover, New York, New York, 1981.

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