Appendix F
Electron Relaxation and Thermalization Times

Spitzer [1] derived an expression for the slowing-down time of test particles (primary electrons in our case) with a velocity $v = \sqrt{2V_p/m}$, where eV_p is the test particle energy, in a population of Maxwellian electrons at a temperature T_e. Spitzer defined the inverse mean velocity of the Maxwellian electron "field particles" in one-dimension as $l_f = \sqrt{m/2kT_e}$. The slowing-down time is then given by

$$\tau_s = \frac{v}{\left(1 + {}^{m}\!/\!{}_{m_f}\right) A_D l_f^2 G(l_f v)}, \tag{F-1}$$

where m is the mass of the test particles, m_f is the mass of the field particles, and A_D is a diffusion constant given by

$$A_D = \frac{8\pi e^4 n_f Z^2 Z_f^2 \ln\Lambda}{m^2}, \tag{F-2}$$

where Z is the charge and $\ln\Lambda$ is the collisionality parameter [2] equal to $23 - \ln\left(n_f^{1/2} / T_e^{3/2}\right)$. The function $G(l_f v)$ is defined as

$$G(x) = \frac{\Phi(x) - x\Phi'(x)}{2x^2}, \tag{F-3}$$

and $\Phi(x)$ is the erf function:

$$\Phi(x) = \frac{2}{\pi^{1/2}} \int_0^x e^{-y^2} dy \,. \tag{F-4}$$

Spitzer gave the values of $G(x)$ in a table, which is plotted in Fig. F-1 and fitted. For $x = 1^2 v$ greater than 1.8, a power function fits best with the relation $G(x) = 0.463x^{-1.957}$.

In our case, the field particles and the test particles have the same mass, which is the electron mass, and charge $Z = e$. The slowing-down time is plotted in Fig. F-2 as a function of the primary particle energy for three representative plasma densities found near the baffle, in the discharge chamber, and near the grids.

For 15-eV primaries in the discharge chamber plasma with an average temperature of 4 eV and a density approaching 10^{12} cm^{-3}, the slowing-down time is about 10^{-6} s. The slowing-down time is also plotted in Fig. F-3 as a function of the plasma density for several values of the primary electron energy, again assuming the plasma has an electron temperature of about 4 eV. As the plasma density increases, the slowing-down time becomes very small ($<10^{-6}$ s). This will lead to rapid thermalization of the primary electrons.

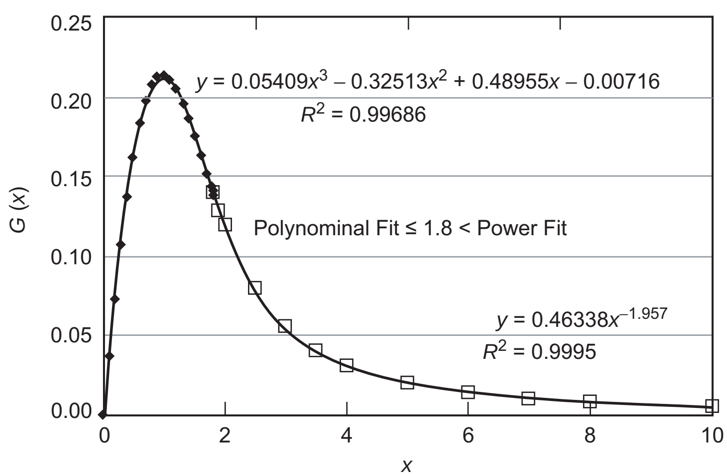

Fig. F-1. Spitzer's *G*(*x*) with fits.

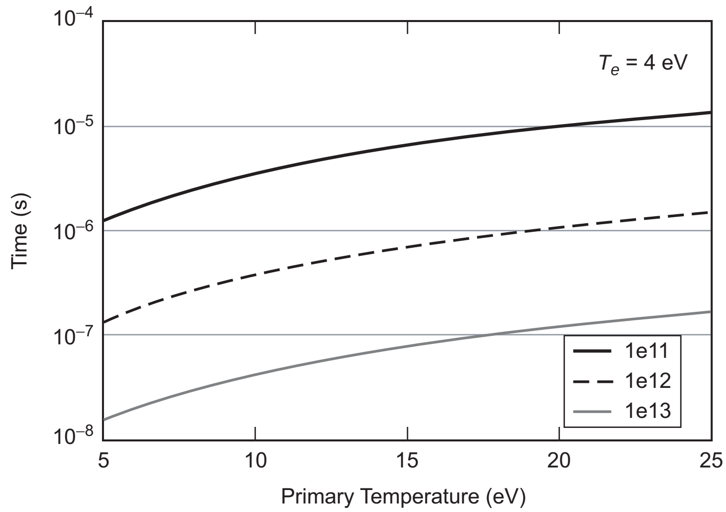

Fig. F-2. Spitzer's slowing-down time as a function of the primary electron energy for three densities of electrons at 4 eV.

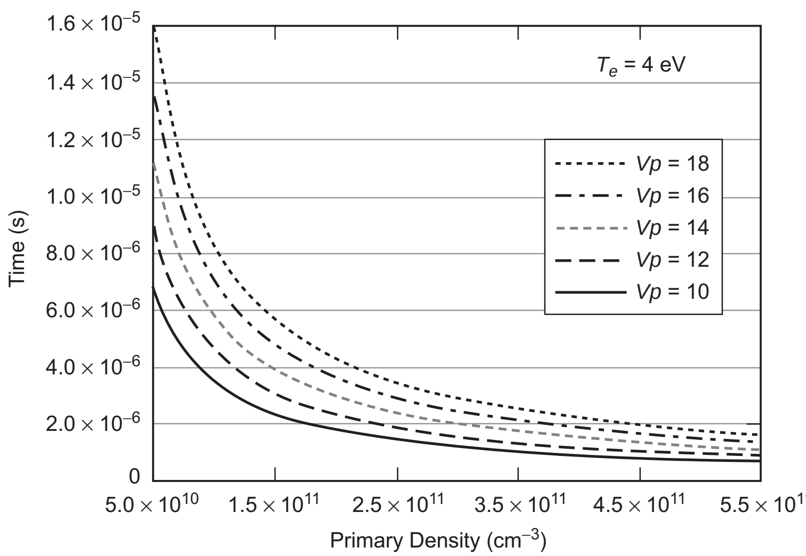

Fig. F-3. Spitzer's slowing-down time as a function of the plasma density with an electron temperature of 4 eV for several primary energies.

For the case of primary electrons with some spread in energy, we can examine the time for the equilibration between that population and the plasma electrons. Assuming that the primaries have a temperature T_1 and the plasma electrons have a temperature T_2, the time for the two populations to equilibrate is

$$\tau_{eq} = \frac{3m^{1/2}\left(kT_1 + kT_2\right)^{3/2}}{8\left(2\pi\right)^{1/2} ne^4 \ln \Lambda} . \tag{F-5}$$

As an example, the slowing time for monoenergetic primaries and primaries with a Maxwellian distribution of energies injected into a 4-eV plasma is shown in Fig. F-4. The slowing time is significantly faster than the equilibration time.

References

[1] L. Spitzer, Jr., *Physics of Fully Ionized Gases*, New York, Interscience, pp. 127–135, 1962.

[2] D. L. Book, *NRL Plasma Formulary*, Naval Research Laboratory, Washington D.C., pp. 33–34, 38, 1987.

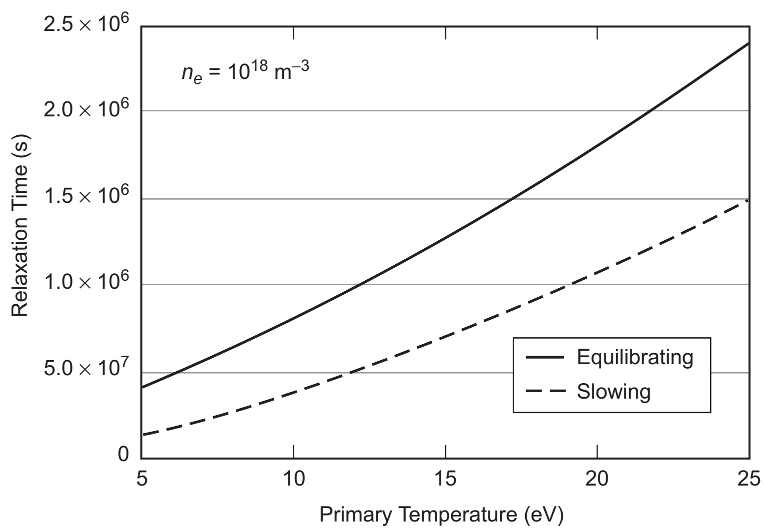

Fig. F-4. Relaxation times of monoenergetic primaries and a Maxwellian primary population in a 4 eV, 10^{18}m^{-3} plasma.