Appendix C
Energy Loss by Electrons

The energy lost from the plasma due to electrons being lost to an anode that is more negative than the plasma is derived. Figure C-1 shows the plasma potential distribution in the negative-going sheath towards the anode wall. The Maxwellian electrons are decelerated and repelled by the sheath potential. To determine the average energy removed from the plasma by each electron, moments of the Maxwellian distribution are taken. The electron current density reaching the wall is given by

$$\begin{aligned} J_e &= en \int_{-\infty}^{\infty} dv_x \int_{-\infty}^{\infty} dv_y \int_{\sqrt{2e\phi/m}}^{\infty} v_z \left(\frac{m}{2\pi k T_e} \right)^{3/2} \exp\left(\frac{-m\left(v_x^2 + v_y^2 + v_z^2\right)}{2kT_e} \right) dv_z \\ &= \frac{1}{4} en \sqrt{\frac{8kT_e}{\pi m}} \exp\left(-\frac{e\phi}{kT_e} \right). \end{aligned} \tag{C-1}$$

The electrons must overcome the sheath potential to reach the wall so the minimum electron speed toward the wall (assumed to be in the z-direction) is $\sqrt{2e\phi/m}$. The plasma electrons lose kinetic energy as they traverse the sheath, so the power flux from plasma is

$$\begin{aligned} P_e &= n \int_{-\infty}^{\infty} dv_x \int_{-\infty}^{\infty} dv_y \int_{\sqrt{2e\phi/m}}^{\infty} v_z \left(\frac{m_e\left(v_x^2 + v_y^2 + v_z^2\right)}{2} \right) \left(\frac{m}{2\pi k T_e} \right)^{3/2} \\ &\quad \times \exp\left(\frac{-m\left(v_x^2 + v_y^2 + v_z^2\right)}{2kT_e} \right) dv_z \\ &= \frac{1}{4} ne \sqrt{\frac{8kT_e}{\pi m}} \left(2\frac{kT_e}{e} + \phi \right) \exp\left(-\frac{e\phi}{kT_e} \right), \end{aligned} \tag{C-2}$$

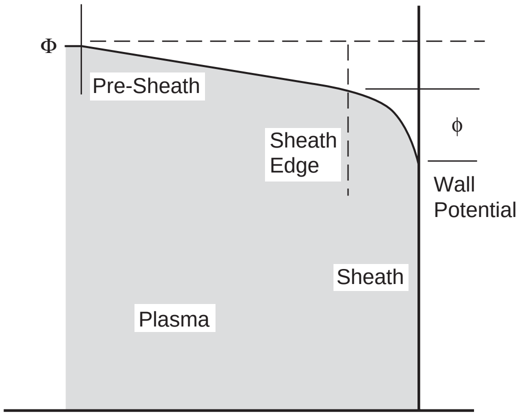

Fig. C-1. Schematic of plasma in contact with the anode wall.

where ϕ is expressed in electron volts (eV). The average energy that an electron removes from the plasma (in eV) is then the ratio of the power per electron to the flux of electrons:

$$E_{\text{ave}} = \frac{P_e}{J_e} = 2\frac{kT_e}{e} + \phi = 2T_{\text{eV}} + \phi\,, \tag{C-3}$$

where T_{eV} is in electron volts (eV). This is the energy removed from the plasma per electron striking the wall through a negative-going sheath.

It should be noted that this energy loss from the plasma per electron is different than the average energy that each electron has when it hits the wall. The flux of electrons hitting the anode wall is the same as analyzed above. The plasma electrons lose kinetic energy as they traverse the sheath; hence, a $-e\phi$ term must be included in the particle energy expression for each electron. The power flux to the insert from plasma electrons is then

$$\begin{aligned} P_e &= n\int_{-\infty}^{\infty} dv_x \int_{-\infty}^{\infty} dv_y \int_{\sqrt{2e\phi/m}}^{\infty} v_z \left(\frac{m_e\left(v_x^2+v_y^2+v_z^2\right)}{2} - e\phi \right)\left(\frac{m}{2\pi kT_e}\right)^{3/2} \\ &\quad \times \exp\left(\frac{-m\left(v_x^2+v_y^2+v_z^2\right)}{2kT_e} \right) dv_z \\ &= \frac{1}{4} ne\sqrt{\frac{8kT_e}{\pi m}} \left(2\frac{kT_e}{e}\right) \exp\left(-\frac{e\phi}{kT_e}\right). \end{aligned} \tag{C-4}$$

The average energy of each electron is then the ratio of the power to the flux:

$$E_{\text{ave}} = \frac{P_e}{J_e} = 2\frac{kT_e}{e} = 2T_{eV} \quad \text{[energy per electron that strikes the wall]} \qquad \text{(C-5)}$$