Appendix B
Gas Flow Unit Conversions and Cathode Pressure Estimates

Conversion between the different systems of flow units is necessary to calculate various parameters used in evaluating thruster performance. Due to the precision required in calculating the thruster performance, it is necessary to carry several significant digits in the constants used to calculate the conversion coefficients, which are obtained from the National Institute of Standards and Technology (NIST) database that can be found on the NIST Web site.

Converting flow in standard cubic centimeters per minute (SCCM) to other flow units for an ideal gas is achieved as follows. A mole of gas at standard pressure and temperature is Avogadro's number ($6.02214179 \times 10^{23}$) of particles at one atmosphere and 0 deg C (273.15 K), which occupies 22.413996 liters. The conversions are

$$\begin{aligned} 1\ \text{sccm} &= \frac{6.02214179 \times 10^{23}[\text{atoms/ mole}]}{22.413996\ [\text{liters/ mole at STP}]*10^{3}[\text{cc/ liter}]*60\ [\text{s/ min}]} \\ &= 4.477962 \times 10^{17}\left[\frac{\text{atoms}}{\text{s}}\right] \end{aligned} \tag{B-1}$$

$$\begin{aligned} 1\ \text{sccm} &= 4.477962 \times 10^{17}\left[\frac{\text{atoms}}{\text{s}}\right]*1.6021765 \times 10^{-19}[\text{coulombs/ charge}] \\ &= 7.174486 \times 10^{-2}\ [\text{equilvalent amperes}] \end{aligned} \tag{B-2}$$

$$1\ \text{sccm} = \frac{10^{-3}[\text{liters}] * 760[\text{torr}]}{60[\text{s/min}]} = 0.01267\ \left[\frac{\text{torr}-1}{\text{s}}\right] \tag{B-3}$$

$$\begin{aligned} 1\ \text{sccm} &= 4.47796 \times 10^{17} \left[\frac{\text{atoms}}{\text{s}}\right] * 1.660539 \times 10^{-27} * M_a * 10^6 \\ &= 7.43583 \times 10^{-4} M_a \left[\frac{\text{mg}}{\text{s}}\right], \end{aligned} \tag{B-4}$$

where M_a is the propellant mass in atomic mass units (AMU).

For xenon, $M_a = 131.293$, and a correction must be made for its compressibility at standard temperature and pressure (STP), which changes the mass flow rate by 0.9931468. Therefore, for xenon,

$$\begin{aligned} 1\ \text{sccm (Xe)} &= \frac{7.17448 \times 10^{-2}}{0.9931468} \\ &= 0.0722399\ [\text{equilvalent amperes}] \\ &= 0.0983009\ [\text{mg/s}]. \end{aligned} \tag{B-5}$$

It is possible to make an estimate of the neutral gas pressure inside of a hollow cathode insert region and in the orifice as a function of the propellant flow rate and cathode temperature, using analytic gas flow equations. While these equations may not be strictly valid in some locations, especially the relatively short orifices found in discharge cathodes, they can still provide an estimate that is usually within 10% to 20% of the actual measured pressures.

In the viscous flow regime, where the transport is due to gas atoms or molecules primarily making collisions with each other rather than walls, the pressure through a cylindrical tube is governed by the Poiseuille law [1,2] modified for compressible gas [3]. The rate at which compressible gas flows through a tube of length l and radius *a* (in moles per second) is given [2] from this law by

$$N_m = \frac{\pi}{8\zeta}\ \frac{a^4}{l}\ \frac{P_a\left(P_1 - P_2\right)}{R_o T} = \frac{\pi}{16\zeta}\ \frac{a^4}{l}\ \frac{P_1^2 - P_2^2}{R_o T}, \tag{B-6}$$

where a is the tube radius, l is the tube length, P_a is the average pressure in the tube given by $(P_1 + P_2)/2$, ζ is the viscosity, P_2 is the downstream pressure at the end of the tube, P_1 is the upstream pressure of the tube, R_o is the universal gas constant, and T is the temperature of the gas. The measured gas flow rate, or the gas throughput, is given by the ideal gas law:

$$Q = P_m V_m = N_m R_o T_m \,, \tag{B-7}$$

where P_m is the pressure and V_m is the volume where the flow is measured for gas at a temperature T_m, and N_m is the mole flow rate. The mole flow rate is then $N_m = P_m V_m / R_o T_m$. Defining $T_r = T / T_m$ and substituting the mole flow rate into Eq. B-1 gives the measured flow as

$$Q = \frac{\pi}{16\zeta} \frac{a^4}{l} \frac{P_1^2 - P_2^2}{T} T_m = \frac{\pi}{16\zeta} \frac{a^4}{l} \frac{P_1^2 - P_2^2}{T_r} \,. \tag{B-8}$$

Putting this in useful units and writing it in terms of a conductance of the tube, which is defined as the gas flow divided by the pressure drop, gives

$$Q = \frac{1.28\, d^4}{\zeta\, T_r\, l} \left(P_1^2 - P_2^2\right), \tag{B-9}$$

where Q is the flow in sccm, η is the viscosity in poises, d is the orifice diameter and l the orifice length in centimeters, and the pressures are in torr. The pressure upstream of the cathode orifice is then

$$P_1 = \left(P_2^2 + \frac{0.78 Q \zeta\, T_r\, l}{d^4} \right)^{1/2} . \tag{B-10}$$

While Eq. (B-10) requires knowledge of the downstream pressure, for this rough estimate it is acceptable to assume $P_2 << P_1$ and neglect this term. For xenon, the viscosity in poises is

$$\zeta = 2.3 \times 10^{-4}\, T_r^{\left(0.71 + 0.29/T_r\right)} \text{ for } T_r > 1 \,, \tag{B-11}$$

where $T_r = T$ (˚K)/289.7. The viscosity in Eq. (B-11) is different than Eq. (6.5-9) because 1 Ns/m^2 = 10 poise. It should be noted that the temperature of the gas in the hollow cathode can exceed the temperature of the cathode by factors of 2 to 4 due to charge-exchange heating with the ions, which then affects the viscosity.

As an example, take the NASA Solar Electric Propulsion Technology Applications Readiness (NSTAR) discharge cathode operating at a nominal flow of 3.7 sccm, with an orifice diameter of 1 mm and the length of the cylindrical section of the orifice 0.75 mm. Assuming the gas in the orifice is 4000 K due to charge-exchange heating and $P_2 = 0$, the upstream pressure is found from Eq. (B-10) to be 6.7 torr. The pressure measured upstream of the cathode tube for this TH15 case is about 8 torr [5]. Correcting for the pressure drop in the insert region (also due to Poiseuille flow), the actual pressure upstream of the orifice plate is about 7.2 torr. The pressure calculated from Eq. (B-10) is low because the downstream pressure is finite (about 2 torr where the barrel section ends) and the bevel region at the output of the orifice has a finite molecular conductance in the collisionless flow regime. In general, it can be assumed that the results of Eq. (B-10) are about 10% low due to these effects. Similar agreement has been found for neutralizer cathodes with straight bore orifices, suggesting that this technique provides reasonable estimates of the pressure in the cathodes.

Finally, once the pressure inside the cathode or in the orifice region entrance is estimated, it is straightforward to calculate the local neutral density from Eq. (2.7-2):

$$n_o = 9.65 \times 10^{24} * \frac{P}{T} \left[\frac{\text{particles}}{\text{m}^3} \right], \tag{B-12}$$

where P is the pressure in torr and T is the gas temperature in kelvins.

References

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