SECTION 7

ALGORITHMS FOR COMPUTING ET – TAI

Contents

7.1 INTRODUCTION

This section gives four algorithms that are used to compute the time difference ET – TAI, where ET is coordinate time of the Solar-System barycentric or local geocentric space-time frame of reference and TAI is International Atomic Time. Section 7.3.1 gives the algorithm for computing ET – TAI at the reception time t_3(TAI) at a tracking station on Earth. The tracking station can be a DSN station or a GPS receiving station. Section 7.3.2 gives the algorithm for computing ET – TAI at the transmission time t_1(ET) at a DSN tracking station on Earth. The algorithm of Section 7.3.3 is used to calculate ET – TAI at the reception time t_3(TAI) at the TOPEX satellite. Finally, the algorithm of Section 7.3.4 is used to compute ET – TAI at the transmission time t_2(ET) at a GPS satellite. These algorithms are evaluated in the spacecraft or quasar light-time solutions, which are described in Section 8. The two algorithms that are evaluated at reception times from the argument t_3(TAI) are iterative and produce all of the position, velocity, and acceleration vectors, which are required at the reception time t_3(ET).

For GPS/TOPEX data, a signal is transmitted from a GPS satellite (semi-major axis $a \approx 26{,}560$ km) and received at the TOPEX satellite ($a \approx 7712$ km) and/or at a GPS receiving station on Earth. In order to process this data, the offset from the station location (at the receiving GPS tracking station on Earth, the receiving TOPEX satellite, and the transmitting GPS satellite) to the phase center (which is the effective point of reception or transmission) must be calculated. These offsets contain a constant offset to the nominal phase center and a variable offset from the nominal phase center to the actual phase center. Section 7.2 introduces the calculation of these offsets and indicates where they are calculated in Sections 1 through 13 of this document.

7.2 PHASE-CENTER OFFSETS FOR GPS/TOPEX DATA

The GPS/TOPEX observables are one-way travel times from a transmitting GPS satellite to the receiving TOPEX satellite or a GPS receiving station on Earth, converted from seconds to kilometers. The exact definitions of these observables are given in Section 13.6. Each of these observables can be a pseudo-range observable or a carrier-phase observable. The pseudo-range signal travels at the group velocity, which is less than the speed of light *c*. The carrier-phase signal travels at the phase velocity, which is greater than *c*. Each GPS satellite transmits signals at the L1-band and L2-band frequencies, which are given by:

$$\begin{aligned} L1 &= 10.23 \text{ MHz} \times 154 = 1575.42 \text{ MHz} \\ L2 &= 10.23 \text{ MHz} \times 120 = 1227.60 \text{ MHz} \end{aligned} \tag{7–1}$$

The pseudo-range and carrier-phase observables come in pairs. Each pair consists of one observable obtained from the L1-band transmitter frequency and a second observable obtained from the L2-band transmitter frequency. Each observable pair is used to construct a weighted average observable, which is free of the effects of charged particles. Let ρ_1(L1) and ρ_1(L2) refer to carrier-phase or pseudo-range observables obtained with the L1-band and L2-band transmitter frequencies. Then, the weighted average ρ_1 of these two observables (which is free from the effects of charged particles) is given by:

$$\rho_1 = A\,\rho_1(\text{L1}) - B\,\rho_1(\text{L2}) \qquad \text{km} \tag{7–2}$$

where the weighting factors *A* and *B* are given by:

$$A = \frac{L1^2}{L1^2 - L2^2} = 2.545,727,780,163,160 \tag{7–3}$$

$$B = \frac{L2^2}{L1^2 - L2^2} = 1.545,727,780,163,160 \tag{7–4}$$

and $A - B = 1$ exactly. The numerators in Eqs. (7–3) and (7–4) cancel the same terms in the denominators of the charged particle effect terms of ρ_1(L1) and ρ_1(L2), respectively. Then, the minus sign in Eq. (7–2) eliminates the effects of charged particles on the weighted average value ρ_1 of the pseudo-range or carrier-phase observable. Since the first-order term of ρ_1(L1) and ρ_1(L2) is the down-leg range r_{23} in kilometers, and $A - B = 1$, the first-order term of the weighted average observable ρ_1 will also be r_{23}.

Since observed values of carrier-phase and pseudo-range observables are calculated as a weighted average using Eqs. (7–2) to (7–4), these same equations must be used in program Regres to calculate the computed values of these observables. However, it is not necessary to calculate computed values of each observable at the L1-band and L2-band frequencies and then compute a weighted average using Eqs. (7–2) through (7–4). Each observable can be computed once from the formulation that is given in Sections 11.5 and 13.6. However, each frequency-dependent term of this formulation must be replaced with a weighted average of the values of the term computed at the L1-band and L2-band frequencies. The weighted average of each frequency-dependent term is calculated from Eqs. (7–2) to (7–4). The frequency-dependent terms are the constant and variable phase-center offsets for the transmitter and receiver and the geometrical phase correction for carrier-phase observables. The formulation for the geometrical phase correction is given in Section 11.5.3.

For a GPS receiving station on Earth, the constant phase-center offset can be included in the calculation of the Earth-fixed position vector of the tracking station. The procedure for doing this is given in Section 7.3.1. Calculation of the constant phase-center offset at the receiving TOPEX satellite is described in Section 7.3.3. Calculation of the constant phase-center offset at the transmitting GPS satellite is described in the algorithm for the spacecraft light-time solution in Section 8.3.6. Calculation of the variable phase-center offsets is described in Section 11.5.4. Calculation of the weighted-average geometrical phase correction for carrier-phase observables is described in Section 11.5.3.

7.3 ALGORITHMS FOR COMPUTING ET – TAI

7.3.1 AT RECEPTION TIME AT TRACKING STATION ON EARTH

The time argument for evaluating the time difference ET − TAI at the reception time at a DSN tracking station or a GPS tracking station on Earth is the reception time t_3(TAI) in International Atomic Time TAI. The algorithm consists of the following steps:

1. Compute an approximate value of ET – TAI in the Solar-System barycentric space-time frame of reference from Eqs. (2–26) to (2–28), where t in Eq. (2–28) is t_3(TAI) in seconds past J2000. From Eq. (2–30), the final value of ET – TAI in the local geocentric space-time frame of reference is 32.184 s. Add these values of ET – TAI to t_3(TAI) to give an approximate value of t_3(ET) in the Solar-System barycentric frame and the final value of t_3(ET) in the geocentric frame. The error in the approximate value of t_3(ET) in the barycentric frame is less than 4×10^{-5} s.

2. At the value of t_3(ET) obtained in Step 1, interpolate the planetary ephemeris for the position, velocity, and acceleration vectors specified in Section 3.1.2.3.1 in the barycentric frame or Section 3.1.2.3.2 in the geocentric frame.

3. Using t_3(ET) from Step 1 as the argument, calculate the geocentric space-fixed position, velocity, and acceleration vectors of the tracking station on Earth from the formulation of Section 5.

3a. If the receiver is a GPS tracking station on Earth, the calculations in Step 3 must be modified to include the constant phase-center offset at the receiver. For a GPS receiving station, the spherical or cylindrical station coordinates are those of a nearby survey benchmark (Section 5.2.1). The Earth-fixed vector offset $\Delta\mathbf{r}_{b_0}$ from the survey benchmark to the tracking station is calculated from Eqs. (5–4) to (5–11) of Section 5.2.2. From Eq. (5–4), the components of $\Delta\mathbf{r}_{b_0}$ are d_N, d_E, and d_U along the north **N**, east **E**, and zenith **Z** unit vectors at the benchmark. The nominal values of d_N, d_E, and d_U represent the displacement from the survey benchmark to a fixed point on the GPS receiving antenna. We must add Δd_N, Δd_E, and Δd_U to d_N, d_E, and d_U, where the increments Δd_N, Δd_E, and Δd_U represent the displacement from the fixed point on the GPS receiving antenna to the nominal location of its phase center.

The offset vector $\Delta\mathbf{r}_{\mathrm{pc}}$ from the fixed reference point on the GPS receiving antenna to the nominal location of its phase center has known components in the antenna X-Y-Z rectangular coordinate system. For reception at the L1-band and L2-band frequencies, let these offset vectors be denoted by:

$$\Delta\mathbf{r}_{\mathrm{pc}}(L1) = \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{L1} \tag{7–5}$$

$$\Delta\mathbf{r}_{\mathrm{pc}}(L2) = \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{L2} \tag{7–6}$$

Let the weighted average of these two vectors be denoted by:

$$\Delta\mathbf{r}_{\mathrm{pc}}(\mathrm{WA}) = \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{\mathrm{WA}} \tag{7–7}$$

where each component of Eq. (7–7) is obtained from the corresponding components of Eqs. (7–5) and (7–6) using Eqs. (7–2) to (7–4). Since the X axis of the GPS receiving antenna is directed north, the components of the vector offset (7–7) along the north, east, and up directions are given by:

$$\begin{aligned} \Delta d_{\mathrm{N}} &= X_{\mathrm{WA}} \\ \Delta d_{\mathrm{E}} &= -Y_{\mathrm{WA}} \\ \Delta d_{\mathrm{U}} &= Z_{\mathrm{WA}} \end{aligned} \tag{7–8}$$

These components must be added to the nominal values d_{N}, d_{E}, and d_{U} and input to program GIN of the ODP. The input values will be used in Eq. (5–4) in program Regres to calculate the vector offset $\Delta\mathbf{r}_{\mathrm{b}_0}$ from the survey benchmark to the weighted average location of the nominal phase center of the GPS receiving antenna.

If this algorithm is being evaluated in the local geocentric space-time frame of reference, it is complete at this point. However, if it is being evaluated in the Solar-System barycentric space-time frame of reference, the remaining steps must be completed.

4. Using position and velocity vectors interpolated from the planetary ephemeris in Step 2 and the geocentric space-fixed position vector of the tracking station on Earth computed in Step 3, calculate ET − TAI in the barycentric frame from Eq. (2–23).

5. Add the value of ET − TAI calculated in Step 4 to t_3(TAI) to give the final value of t_3(ET) in the barycentric frame.

6. Map the position and velocity vectors obtained from the planetary ephemeris in Step 2 and the geocentric space-fixed position and velocity vectors of the tracking station on Earth obtained in Step 3 from the approximate value of t_3(ET) obtained in Step 1 to the final value of t_3(ET) obtained in Step 5. Let

t_3 = final value of t_3(ET) from Step 5
t_3^* = approximate value of t_3(ET) from Step 1
δt = $t_3 - t_3^*$

Then, each position vector can be mapped with a quadratic Taylor series:

$$\mathbf{r}(t_3) = \mathbf{r}\left(t_3^*\right) + \dot{\mathbf{r}}\left(t_3^*\right)\delta t + \tfrac{1}{2}\ddot{\mathbf{r}}\left(t_3^*\right)\left(\delta t\right)^2 \tag{7–9}$$

and each velocity vector can be mapped with a linear Taylor series:

$$\dot{\mathbf{r}}(t_3) = \dot{\mathbf{r}}\left(t_3{}^*\right) + \ddot{\mathbf{r}}\left(t_3{}^*\right)\delta t \tag{7–10}$$

Also, map each component of the 3 x 3 body-fixed to space-fixed transformation matrix T_E for the Earth and true sidereal time θ, which are calculated in Step 3, using quadratic and linear Taylor series, respectively.

7. Using the mapped position and velocity vectors from Step 6, recalculate ET – TAI in the barycentric frame from Eq. (2–23).

7.3.2 AT TRANSMISSION TIME AT TRACKING STATION ON EARTH

The time argument for calculating the time difference ET – TAI at the transmission time at a DSN tracking station on Earth is the transmission time t_1(ET), which is available from the light-time solution. The algorithm contains two steps:

1. Given position and velocity vectors obtained at the transmission time t_1(ET) in the spacecraft light-time solution, calculate ET – TAI at the tracking station from Eq. (2–23) in the Solar-System barycentric frame of reference. In the local geocentric frame of reference, calculate ET – TAI at the tracking station from Eq. (2–30).

2. Subtract ET – TAI from t_1(ET) to give t_1(TAI).

7.3.3 AT RECEPTION TIME AT TOPEX SATELLITE

The time argument for evaluating the time difference ET – TAI at the reception time at the TOPEX satellite is the reception time t_3(TAI) in International Atomic Time TAI. The algorithm consists of the following steps:

1. Compute an approximate value of ET – TAI in the Solar-System barycentric frame from Eqs. (2–26) to (2–28), where t in Eq. (2–28) is t_3(TAI) in seconds past J2000. From Eq. (2–31), the approximate value

of ET – TAI in the geocentric frame is 32.184 s. Add the approximate value of ET – TAI to t_3(TAI) to give an approximate value of t_3(ET).

2. At the value of t_3(ET) obtained in Step 1, interpolate the planetary ephemeris for the position, velocity, and acceleration vectors specified in Section 3.1.2.3.1 in the barycentric frame or Section 3.1.2.3.2 in the geocentric frame.

3. If the ODP is operating in the Solar-System barycentric frame of reference or the local geocentric frame of reference, program PV integrates the ephemeris of the TOPEX satellite in that frame of reference. At the value of t_3(ET) obtained in Step 1, interpolate the TOPEX satellite ephemeris for the geocentric position, velocity, and acceleration vectors of the TOPEX satellite. These vectors are for the center of mass of the TOPEX satellite. Let **X**, **Y**, and **Z** be unit vectors aligned with the x, y, and z axes of the spacecraft-fixed coordinate system of the TOPEX satellite, directed outward from the origin of the coordinate system. Interpolation of the PV file for the TOPEX satellite at t_3(ET) gives the space-fixed rectangular components of the unit vectors **X**, **Y**, and **Z** referred to the mean Earth equator and equinox of J2000. It also gives the x, y, and z rectangular components referred to the TOPEX-fixed rectangular coordinate system of the weighted-average location of the nominal phase center of the TOPEX satellite relative to the center of mass of the TOPEX satellite. Given this information, calculate the space-fixed vector from the center of mass of the TOPEX satellite to the weighted-average location of the nominal phase center of the TOPEX satellite from:

$$\Delta\mathbf{r} = x\,\mathbf{X} + y\,\mathbf{Y} + z\,\mathbf{Z} \qquad \text{km} \qquad (7\text{–}11)$$

Add this vector offset to the geocentric space-fixed position vector of the center of mass of the TOPEX satellite to give the geocentric space-fixed position vector of the weighted-average location of the nominal phase center of the TOPEX satellite.[1] Save the unit vectors **X**, **Y**, and **Z**.

The x, y, and z components in Eq. (7–11) must be calculated by the user and input to program GIN of the ODP. Program PV will read them from the GIN file and place them on the PV file for the TOPEX satellite. The x, y, and z components are calculated as the sum of three terms:

$$x = x_{\text{geom}} + x_{\text{pc}} - x_{\text{cm}} \qquad x \to y, z \tag{7–12}$$

where x_{geom} is from the origin of the TOPEX satellite coordinate system to a fixed point on or near the GPS antenna (which receives the signal transmitted by a GPS satellite), x_{pc} is the offset from this point to the nominal phase center of the GPS antenna, and x_{cm} is from the origin of the TOPEX satellite coordinate system to the center of mass of the TOPEX satellite. This coordinate is a slowly varying function of time, but must be fixed during a given execution of the ODP. The components x_{pc}, y_{pc}, and z_{pc} of the phase center offset are known at the L1-band frequency and at the L2-band frequency. For each component, the weighted average of the L1-band value and the L2-band value must be computed from Eqs. (7–2) to (7–4) and used in that component of Eq. (7–12).

4. Using position and velocity vectors obtained in Steps 2 and 3, calculate ET – TAI at the TOPEX satellite from Eqs. (2–23) to (2–25) in the barycentric frame and from Eqs. (2–24) and (2–31) in the geocentric frame.

[1]It is not necessary to calculate $\Delta\dot{\mathbf{r}}$ and add it to the satellite velocity vector (for the TOPEX or a GPS satellite). The slight change in the satellite velocity vector would affect the computed down-leg range through the effect of $\Delta\dot{\mathbf{r}}$ on the computed time transformations by less than 1 mm. The change $\Delta\dot{\mathbf{r}}$ to the satellite velocity vector would affect mapped satellite position vectors at the reception time t_3 and at the transmission time t_2 by less than 0.1 mm.

5. Add the value of ET – TAI calculated in Step 4 to t_3(TAI) to give the final value of t_3(ET).

6. Map the position and velocity vectors obtained in Steps 2 and 3 at the approximate value of t_3(ET) obtained in Step 1 to the final value of t_3(ET) obtained in Step 5 using Eqs. (7–9) and (7–10).

7. Using the mapped position and velocity vectors from Step 6, recalculate ET – TAI from Eqs. (2–23) to (2–25) in the barycentric frame and from Eqs. (2–24) and (2–31) in the geocentric frame.

7.3.4 AT TRANSMISSION TIME AT A GPS SATELLITE

The time argument for calculating the time difference ET – TAI at the transmission time at a GPS satellite is the transmission time t_2(ET), which is available from the light-time solution. The algorithm contains two steps:

1. Given position and velocity vectors obtained from the spacecraft light-time solution at the transmission time t_2(ET) at the GPS satellite, calculate ET – TAI at the GPS satellite from Eqs. (2–23) to (2–25) in the barycentric frame and from Eqs. (2–24) and (2–31) in the geocentric frame.

2. Subtract ET – TAI from t_2(ET) to give t_2(TAI).