

SECTION 3
PLANETARY EPHEMERIS, SMALL-BODY
EPHEMERIS, AND SATELLITE
EPHEMERIDES

Contents

3.1	Planetary Ephemeris and Small-Body Ephemeris.....	3-3
3.1.1	Description	3-3
3.1.2	Position, Velocity, and Acceleration Vectors Interpolated From the Planetary Ephemeris and a Small-Body Ephemeris	3-5
3.1.2.1	Position, Velocity, and Acceleration Vectors Which Can Be Interpolated From the Planetary Ephemeris and a Small-Body Ephemeris	3-5
3.1.2.2	Gravitational Constants on the Planetary Ephemeris and a Small-Body Ephemeris	3-7
3.1.2.3	Vectors Interpolated From the Planetary Ephemeris and a Small-Body Ephemeris in a Spacecraft Light-Time Solution and in a Quasar Light-Time Solution	3-8
3.1.2.3.1	Spacecraft or Quasar Light- Time Solution in the Solar- System Barycentric Frame of Reference	3-8

SECTION 3

3.1.2.3.2	Spacecraft Light-Time Solution in the Geocentric Frame of Reference.....	3-10
3.1.3	Partial Derivatives of Position Vectors Interpolated From the Planetary Ephemeris and a Small-Body Ephemeris With Respect to Reference Parameters.....	3-11
3.1.3.1	Required Partial Derivatives.....	3-11
3.1.3.2	The Planetary Partial File.....	3-12
3.1.3.3	Equations for the Required Partial Derivatives of Position Vectors With Respect to Reference Parameters	3-14
3.1.4	Correcting the Planetary Ephemeris.....	3-17
3.2	Satellite Ephemerides.....	3-17
3.2.1	Description	3-17
3.2.2	Position, Velocity, and Acceleration Vectors Interpolated From Satellite Ephemerides.....	3-18
3.2.2.1	Interpolation of Satellite Ephemerides.....	3-18
3.2.2.2	Vectors Interpolated From Satellite Ephemerides	3-19
3.2.3	Partial Derivatives of Position Vectors Interpolated From Satellite Ephemerides.....	3-20

3.1 PLANETARY EPHEMERIS AND SMALL-BODY EPHEMERIS

3.1.1 DESCRIPTION

Interpolation of the planetary ephemeris produces the position (P), velocity (V), and acceleration (A) vectors of the major celestial bodies of the Solar System. The P, V, and A vectors of the Sun, Mercury, Venus, the Earth-Moon barycenter, and the barycenters of the planetary systems Mars, Jupiter, Saturn, Uranus, Neptune, and Pluto are relative to the Solar-System barycenter. The P, V, and A vectors of the Moon are relative to the Earth. All of these vectors have rectangular components referred to the space-fixed coordinate system, which is nominally aligned with the mean Earth equator and equinox of J2000. The time argument is seconds of coordinate time (ET) past J2000 in the Solar-System barycentric space-time frame of reference.

The planetary ephemeris is obtained from a simultaneous numerical integration of the equations of motion for the nine planets, the Moon, and the lunar physical librations. The P, V, and A vectors of the Sun relative to the Solar-System barycenter are calculated from the relativistic definition of the center of mass of the Solar System and its time derivatives. This method for performing the numerical integration is an iterative process. A detailed description of the process of creating the planetary ephemeris is given in Newhall *et al.* (1983). The values of the parameters needed to perform the numerical integration are obtained by fitting computed values of the observations of the Solar-System bodies to the corresponding observed values in a least squares sense. The equations of motion are given in Newhall *et al.* (1983). The observations include optical data (transit and photographic), radar ranging, spacecraft ranging, and lunar laser ranging. The observations and the parameters of the fit are discussed in great detail in Standish (1990) and also in Newhall *et al.* (1983).

The numerical integration produces a file of positions, velocities, and accelerations at equally spaced times for each component being integrated. This information is represented by using Chebyshev polynomials as described in

SECTION 3

detail in Newhall (1989). Each of the three components of the position of the nine planets and the Sun relative to the Solar-System barycenter and the Moon relative to the Earth are represented by an N th-degree expansion in Chebyshev polynomials. Table 1 of Newhall (1989) gives the polynomial degree N and the time span or granule length of the polynomial used for each of the eleven ephemeris bodies. The polynomial degree N varies from 5 to 13, and the granule length varies from 4 to 32 days. Velocity and acceleration components are obtained by replacing the Chebyshev polynomials in the N th-degree expansions in Chebyshev polynomials with their first- and second-time derivatives.

The various celestial reference frames are all nominally aligned with the mean Earth equator and equinox of J2000. The celestial reference frame defined by the planetary ephemeris (the planetary ephemeris frame, PEF) can have a slightly different orientation for each planetary ephemeris. The right ascensions and declinations of quasars and the geocentric space-fixed position vectors of tracking stations on Earth are referred to the radio frame (RF). This particular celestial reference frame is maintained by the International Earth Rotation Service (IERS). The rotation from the PEF to the RF is modelled in the ODP. This frame-tie rotation matrix is a function of solve-for rotations about the three axes of the space-fixed coordinate system as described in detail in Section 5.3. The three rotation angles are different for each planetary ephemeris. However, for any DE400-series planetary ephemeris (*e.g.*, DE405), the PEF is the RF, and the three frame-tie rotation angles are zero. The space-fixed coordinate system adopted for use in the ODP is the PEF for the planetary ephemeris being used. The spacecraft ephemeris is numerically integrated in the PEF. It will be seen in Section 5 that geocentric space-fixed position vectors of Earth-fixed tracking stations are rotated from the RF to the PEF using the frame-tie rotation matrix. Also, Section 8.4 shows that space-fixed unit vectors to quasars are also rotated from the RF to the PEF.

Heliocentric space-fixed P , V , and A vectors of asteroids and comets are obtained by interpolating the small-body ephemeris. The celestial reference frame of the small-body ephemeris is assumed to be that of the planetary ephemeris being used by the ODP. Adding P , V , and A vectors of the Sun

relative to the Solar-System barycenter, obtained by interpolating the planetary ephemeris, gives space-fixed P, V, and A vectors of asteroids and comets relative to the Solar-System barycenter.

**3.1.2 POSITION, VELOCITY, AND ACCELERATION VECTORS
INTERPOLATED FROM THE PLANETARY EPHEMERIS AND A
SMALL-BODY EPHEMERIS**

**3.1.2.1 Position, Velocity, and Acceleration Vectors Which Can Be
Interpolated From the Planetary Ephemeris and a Small-Body
Ephemeris**

Let

$$\mathbf{r}_a^b, \dot{\mathbf{r}}_a^b, \text{ and } \ddot{\mathbf{r}}_a^b$$

denote position, velocity, and acceleration vectors of point a relative to point b . The planetary ephemeris can be interpolated for the position (P), velocity (V), and acceleration (A) vectors of the nine planets (P) relative to the Solar-System barycenter (C):

$$\mathbf{r}_P^C, \dot{\mathbf{r}}_P^C, \text{ and } \ddot{\mathbf{r}}_P^C$$

where P can be Mercury (Me), Venus (V), the Earth-Moon barycenter (B), and the barycenters of the planetary systems Mars (Ma), Jupiter (J), Saturn (Sa), Uranus (U), Neptune (N), and Pluto (Pl). The planetary ephemeris can also be interpolated for the P, V, and A vectors of the Sun (S) relative to the Solar-System barycenter (C):

$$\mathbf{r}_S^C, \dot{\mathbf{r}}_S^C, \text{ and } \ddot{\mathbf{r}}_S^C$$

and the Moon (M) relative to the Earth (E).

$$\mathbf{r}_M^E, \dot{\mathbf{r}}_M^E, \text{ and } \ddot{\mathbf{r}}_M^E$$

SECTION 3

These latter vectors can be broken down into their component parts:

$$\mathbf{r}_B^E = \frac{1}{1+\mu} \mathbf{r}_M^E \quad \mathbf{r} \rightarrow \dot{\mathbf{r}}, \ddot{\mathbf{r}} \quad (3-1)$$

and

$$\mathbf{r}_M^B = \frac{\mu}{1+\mu} \mathbf{r}_M^E \quad \mathbf{r} \rightarrow \dot{\mathbf{r}}, \ddot{\mathbf{r}} \quad (3-2)$$

where B is the Earth-Moon barycenter,

$$\mu = \frac{\mu_E}{\mu_M} \quad (3-3)$$

and

$$\mu_E, \mu_M = \text{gravitational constants of the Earth and Moon, km}^3/\text{s}^2.$$

The small-body ephemeris can be interpolated for the P, V, and A vectors of asteroids and comets (P) relative to the Sun (S):

$$\mathbf{r}_P^S, \dot{\mathbf{r}}_P^S, \text{ and } \ddot{\mathbf{r}}_P^S$$

The time argument for interpolating the planetary ephemeris and a small-body ephemeris for the vectors listed above is seconds of coordinate time (denoted as ET) past J2000 in the Solar-System barycentric space-time frame of reference. The planetary and small-body ephemerides use Chebyshev polynomials to represent the rectangular components of the above vectors in kilometers and seconds of coordinate time ET. The planetary ephemeris contains the scale factor AU , which is the number of kilometers per astronomical unit, and the Earth-Moon mass ratio μ given by Eq. (3-3). Eqs. (3-1) and (3-2) are evaluated in the interpolator for the planetary ephemeris. Solutions for planetary and small-body ephemerides are obtained in astronomical units and days of 86400 s of coordinate time ET. Each solution for a planetary ephemeris includes an estimate for the scale factor AU . It is used to convert solutions for planetary

PLANETARY, SMALL-BODY, AND SATELLITE EPHEMERIDES

and small-body ephemerides from astronomical units and days to kilometers and seconds.

The vectors interpolated from the planetary and small-body ephemerides have rectangular components referred to the space-fixed coordinate system of the planetary ephemeris, which is nominally aligned with the mean Earth equator and equinox of J2000. The misalignment of the planetary ephemeris frame (PEF) from the radio frame (RF) is accounted for in the ODP as described above in the penultimate paragraph of Section 3.1.1.

The planetary ephemeris represents the P , V , and A vectors of Mercury, Venus, the Earth-Moon barycenter, the barycenters of the planetary systems Mars through Pluto, the Sun, the Earth, and the Moon relative to the Solar-System barycenter. The interpolator adds or subtracts the various vectors listed above to obtain the P , V , and A vectors of any one of these points relative to any other of these points. It specifically calculates the P , V , and A vectors for the points specified by the user relative to the center that he specifies.

As stated in Section 3.1.1, adding Solar-System barycentric P , V , and A vectors of the Sun, obtained by interpolating the planetary ephemeris, to heliocentric P , V , and A vectors of an asteroid or a comet, obtained by interpolating a small-body ephemeris, gives Solar-System barycentric P , V , and A vectors of an asteroid or a comet.

3.1.2.2 Gravitational Constants on the Planetary Ephemeris and a Small-Body Ephemeris

The planetary ephemeris contains the gravitational constants for Mercury; Venus; the Earth, the Moon, and their sum; the planetary systems Mars through Pluto; and the Sun. From Section 2.3.1.1, each of these gravitational constants μ is the product of the universal gravitational constant G and the rest mass m of the body or system of bodies. The gravitational constants are given in astronomical units cubed per day squared and in kilometers cubed per second squared, where the latter set (which is used in the ODP) is obtained from the former set by multiplying by $AU^3/(86400)^2$. The gravitational constant of the Sun in

SECTION 3

astronomical units cubed per day squared is the square of the Gaussian gravitational constant, which defines the length of one astronomical unit. The gravitational constant of the Sun (μ_S) in kilometers cubed per second squared is thus a function of the value of the scale factor AU . The ODP user is not allowed to estimate the values of μ_S and AU , which are obtained from the planetary ephemeris.

The small-body ephemeris contains the gravitational constants (in units of kilometers cubed per second squared) for all of the bodies (asteroids and comets) contained in the file.

3.1.2.3 Vectors Interpolated From the Planetary Ephemeris and a Small-Body Ephemeris in a Spacecraft Light-Time Solution and in a Quasar Light-Time Solution

For a spacecraft light-time solution, the planetary ephemeris is interpolated at the ET values of the reception time t_3 at a tracking station on Earth or an Earth satellite, the reflection time or transmission time t_2 at the spacecraft, and (if the spacecraft is not the transmitter) at the transmission time t_1 at a tracking station on Earth or an Earth satellite. For a quasar light-time solution, the planetary ephemeris is interpolated at the ET values of the reception time t_1 of the quasar wavefront at receiver 1 and the reception time t_2 of the quasar wavefront at receiver 2. Receiver 1 and receiver 2 can each be a tracking station on Earth or an Earth satellite.

For a spacecraft light-time solution, the small-body ephemeris may be interpolated at the ET value of the reflection time or transmission time t_2 at the spacecraft.

3.1.2.3.1 Spacecraft or Quasar Light-Time Solution in the Solar-System Barycentric Frame of Reference

For either of these light-time solutions, interpolate the planetary ephemeris and the small-body ephemeris for the P, V, and A vectors of the following bodies or planetary system centers of mass at the specified times. The

PLANETARY, SMALL-BODY, AND SATELLITE EPHEMERIDES

name of a planet (other than the Earth) implies the barycenter of the planetary system. If the origin of these vectors is not specified, it is the Solar-System barycenter (C).

1. The Sun, Jupiter, and Saturn at each interpolation epoch.
2. If the relativistic light-time delay (see Section 8) is calculated for Mercury, Venus, the Earth, the Moon, or the barycenters of the planetary systems Mars, Uranus, Neptune, or Pluto, interpolate vectors for each of these bodies at each interpolation epoch.
3. The participant central body (PCB) is the intermediate body between the participant (*e.g.*, a tracking station or the spacecraft) and the Solar-System barycenter. If the PCB is the Earth, which it will be at t_3 and t_1 for a spacecraft light-time solution and at t_1 and t_2 for a quasar light-time solution, interpolate vectors for the Earth-Moon barycenter, the Earth, the Moon, and the geocentric Moon and its component parts (Eqs. 3–1 and 3–2).
4. Interpolate vectors for the PCB at t_2 for a spacecraft light-time solution. The PCB is the center of integration (COI) for the spacecraft ephemeris, or it is the body on which a landed spacecraft is resting. If the COI is the Sun, Mercury, Venus, the Earth, the Moon, or the barycenter of one of the planetary systems Mars through Pluto, interpolate vectors for this point. If the COI is the planet or a satellite of an outer planet system, interpolate vectors for the barycenter of this planetary system. If a landed spacecraft is on Mercury, Venus, or the Moon, obtain vectors for this body. If a landed spacecraft is on the planet or a satellite of one of the outer planet systems, interpolate vectors for the barycenter of this planetary system. If the PCB is the Earth or the Moon, interpolate all of the vectors listed above in item 3.

If the center of integration for the spacecraft ephemeris or the body upon which a landed spacecraft is resting is an asteroid or a comet,

SECTION 3

interpolate the small-body ephemeris for the heliocentric vectors of the asteroid or comet at t_2 . Add the Solar-System barycentric vectors of the Sun (available from Step 1) to give the Solar-System barycentric vectors of the asteroid or comet.

5. At t_2 for a one-way doppler (F_1) data point or a one-way wideband or narrowband spacecraft interferometry observable (IWS or INS) (see Section 13), interpolate vectors for the Sun, Mercury, Venus, the Earth, the Moon, and the barycenters of the planetary systems Mars through Pluto. Program PV interpolates the small-body ephemeris for the vectors of the small bodies (up to ten of them) specified in the input variables XBNUM and XBNAM. It calculates the acceleration of the spacecraft due to these small bodies. At t_2 for F_1 , IWS , or INS , program Regres should interpolate the small-body ephemeris for the heliocentric vectors of the small bodies specified in the input arrays XBNUM and XBNAM. If there are more names in these arrays than are on the small-body ephemeris, obtain vectors for the latter set. It is up to the ODP user to make sure that the small-body ephemeris that Regres is reading contains the asteroid or comet that the spacecraft is encountering and that the number and name of this body are contained in the input arrays XBNUM and XBNAM. Convert the heliocentric vectors for the small bodies to Solar-System barycentric vectors as described above in item 4.

3.1.2.3.2 Spacecraft Light-Time Solution in the Geocentric Frame of Reference

The geocentric light-time solution (see Section 8) is used to process GPS/TOPEX data (see Section 13). The only quantities required from the planetary ephemeris are the geocentric position vectors of the Sun and the Moon at the reception time t_3 if the receiver is a tracking station on Earth. They are used to compute the displacement of the station due to solid Earth tides. If this one-way light-time solution is ever extended to the round-trip mode, these same vectors will be required at the transmission time t_1 .

3.1.3 PARTIAL DERIVATIVES OF POSITION VECTORS INTERPOLATED FROM THE PLANETARY EPHEMERIS AND A SMALL-BODY EPHEMERIS WITH RESPECT TO REFERENCE PARAMETERS

3.1.3.1 Required Partial Derivatives

The ODP calculates partial derivatives of the computed values of the observables with respect to the parameter vector \mathbf{q} . The parameter vector \mathbf{q} includes solve-for parameters and consider parameters. The former parameters are those whose values are estimated in the filter. The latter parameters are those whose uncertainties are considered when calculating the uncertainties in the values of the estimated parameters.

Calculation of the partial derivatives of the computed values of the observables with respect to \mathbf{q} requires the partial derivatives of certain position vectors interpolated from the planetary and small-body ephemerides with respect to \mathbf{q} . The partial derivative of the Solar-System barycentric position vector of the Earth with respect to \mathbf{q} is required at t_3 and t_1 for a spacecraft light-time solution and at t_1 and t_2 for a quasar light-time solution. Also required is the partial derivative of the Solar-System barycentric position vector of the PCB at t_2 for a spacecraft light-time solution (see Section 3.1.2.3.1, item 4). These partial derivatives are non-zero only for the so-called reference parameters. For the planetary ephemeris, they are the Brouwer and Clemence Set III orbital element corrections for the nine planetary ephemerides and for the geocentric lunar ephemeris, the *AU* scaling factor for the planetary ephemeris (which can be considered but not estimated), and the gravitational constants for the Earth (μ_E) and the Moon (μ_M). For a small-body ephemeris, the reference parameters are the Brouwer and Clemence Set III orbital element corrections or the Keplerian orbital parameters e , q , T_p , Ω , ω and i , dynamical parameters such as the cometary nongravitational parameters A_1 and A_2 , and the *AU* scaling factor from the planetary ephemeris.

The Set III partials for the planetary ephemeris are obtained by interpolating the planetary partials file. The contents of this file and the

SECTION 3

procedure used to create it are described in Section 3.1.3.2. The partial derivatives of the heliocentric position vector of an asteroid or a comet with respect to the Set III orbital element corrections or the Keplerian orbital parameters, and dynamical parameters A_1 and A_2 are obtained by interpolating the small-body partials file. Section 3.1.3.3 gives the equations for the required partial derivatives of position vectors obtained from the planetary ephemeris and a small-body ephemeris with respect to the reference parameters.

3.1.3.2 The Planetary Partial File

The planetary partials file can be interpolated for the partial derivatives of the position (P) and velocity (V) vectors of the nine planets (P) relative to the Sun (S) with respect to the six Brouwer and Clemence Set III orbital element corrections ($\Delta\mathbf{E}$):

$$\frac{\partial \mathbf{r}_P^S}{\partial \Delta\mathbf{E}'} \quad \frac{\partial \dot{\mathbf{r}}_P^S}{\partial \Delta\mathbf{E}}$$

where P can be Mercury (Me), Venus (V), the Earth-Moon barycenter (B), and the barycenters of the planetary systems Mars (Ma), Jupiter (J), Saturn (Sa), Uranus (U), Neptune (N), and Pluto (Pl). The planetary partials file can also be interpolated for the partial derivatives of the P and V vectors of the Moon (M) relative to the Earth (E) with respect to $\Delta\mathbf{E}$:

$$\frac{\partial \mathbf{r}_M^E}{\partial \Delta\mathbf{E}'} \quad \frac{\partial \dot{\mathbf{r}}_M^E}{\partial \Delta\mathbf{E}}$$

The time argument for interpolating the planetary partials file is seconds of coordinate time (ET) past J2000 in the Solar-System barycentric space-time frame of reference. The P and V vectors in the interpolated partial derivatives have rectangular components referred to the space-fixed coordinate system nominally aligned with the mean Earth equator and equinox of J2000 (*i.e.*, the planetary ephemeris frame) and have units of kilometers and kilometers per second.

PLANETARY, SMALL-BODY, AND SATELLITE EPHEMERIDES

The Brouwer and Clemence Set III orbital element corrections $\Delta\mathbf{E}$ are six parameters that represent corrections to the osculating orbital elements at the osculation epoch $t_0(\text{ET})$:

$$\Delta\mathbf{E} = \begin{bmatrix} \Delta a/a \\ \Delta e \\ \Delta M_0 + \Delta w \\ \Delta p \\ \Delta q \\ e \Delta w \end{bmatrix} \quad \text{rad} \quad (3-4)$$

where

- a = semimajor axis of osculating elliptical orbit
- e = eccentricity
- M_0 = value of mean anomaly at osculation epoch $t_0(\text{ET})$
- $\Delta p, \Delta q, \Delta w$ = right-handed rotations of the orbit about the \mathbf{P} , \mathbf{Q} , and \mathbf{W} axes, respectively, where \mathbf{P} is directed from the focus to perifocus, \mathbf{Q} is $\pi/2$ rad ahead of \mathbf{P} in the orbital plane, and $\mathbf{W} = \mathbf{P} \times \mathbf{Q}$

The partial derivatives of position and velocity vectors with respect to Set III orbital element corrections which are listed above and are contained in the planetary partials file are calculated from the following equation:

$$\frac{\partial \mathbf{X}(t)}{\partial \Delta\mathbf{E}} = U(t, t_0) \frac{\partial \mathbf{X}(t_0)}{\partial \Delta\mathbf{E}} \quad (3-5)$$

where

$$\mathbf{X} \equiv \begin{bmatrix} \mathbf{r} \\ \dot{\mathbf{r}} \end{bmatrix} \quad (3-6)$$

and

SECTION 3

$$U(t, t_0) = \frac{\partial \mathbf{X}(t)}{\partial \mathbf{X}(t_0)} \quad (3-7)$$

This 6×6 matrix U is obtained by numerical integration. The partial derivatives of \mathbf{r} and $\dot{\mathbf{r}}$ at the osculation epoch t_0 with respect to the Set III orbital element corrections $\Delta \mathbf{E}$ at this epoch are calculated from Eqs. (115) to (148) of Moyer (1971) using \mathbf{r} and $\dot{\mathbf{r}}$ interpolated from the planetary ephemeris at the osculation epoch $t_0(\text{ET})$. Note that these vectors are Sun-centered for the nine planetary ephemerides. For the ten ephemerides on the planetary partials file, the osculation epoch $t_0(\text{ET})$ is June 28, 1969, 0^h (JD 2440400.5).

Future versions of the planetary partials file will probably be generated from finite difference partial derivatives instead of numerically integrated partial derivatives.

3.1.3.3 Equations for the Required Partial Derivatives of Position Vectors With Respect to Reference Parameters

This section gives the equations for the partial derivatives of the position vectors (measured in kilometers) of the Earth (E), the Moon (M), a planet (P) (which can be Mercury, Venus, and the barycenters of the planetary systems Mars through Pluto), the Sun (S), and an asteroid or a comet (P) relative to the Solar-System barycenter (C) with respect to the reference parameters. These partial derivatives are calculated at the epochs (t_1 , t_2 , or t_3) specified in Section 3.1.3.1.

The partial derivative of the Solar-System barycentric position vector of a body b (which can be the Earth, the Moon, a planet, the Sun, an asteroid, or a comet) with respect to the AU scaling factor is given by:

$$\frac{\partial \mathbf{r}_b^C}{\partial AU} = \frac{\mathbf{r}_b^C}{AU} \quad (3-8)$$

where the position vectors are in kilometers and AU is kilometers/astronomical unit.

The following equations give the partial derivatives of the required position vectors with respect to Set III orbital element corrections (or the alternate Keplerian orbital parameters for the orbit of an asteroid or a comet), the gravitational constants of the Earth and the Moon, and the cometary nongravitational parameters A_1 and A_2 . Only the non-zero partials are given. The high-level equations for the partials of the Solar-System barycentric position vectors of the Earth and the Moon are given by:

$$\frac{\partial \mathbf{r}_E^C}{\partial \mathbf{q}} = \frac{\partial \mathbf{r}_B^C}{\partial \mathbf{q}} - \frac{\partial \mathbf{r}_B^E}{\partial \mathbf{q}} \quad (3-9)$$

$$\frac{\partial \mathbf{r}_M^C}{\partial \mathbf{q}} = \frac{\partial \mathbf{r}_B^C}{\partial \mathbf{q}} + \frac{\partial \mathbf{r}_M^B}{\partial \mathbf{q}} \quad (3-10)$$

where

$$\frac{\partial \mathbf{r}_B^C}{\partial \mathbf{q}} = \frac{\partial \mathbf{r}_B^S}{\partial \Delta E} \quad (3-11)$$

which is interpolated from the planetary partials file. The non-zero partials for the last term of Eq. (3-9) are given by:

$$\frac{\partial \mathbf{r}_B^E}{\partial \Delta E_M} = \frac{1}{1 + \mu} \frac{\partial \mathbf{r}_M^E}{\partial \Delta E_M} \quad (3-12)$$

where μ is given by Eq. (3-3) and the Set III partials for the geocentric lunar ephemeris are obtained from the planetary partials file.

$$\frac{\partial \mathbf{r}_B^E}{\partial \mu_E} = - \frac{\mathbf{r}_M^E}{(1 + \mu)^2 \mu_M} \quad (3-13)$$

$$\frac{\partial \mathbf{r}_B^E}{\partial \mu_M} = \frac{\mu \mathbf{r}_M^E}{(1 + \mu)^2 \mu_M} \quad (3-14)$$

SECTION 3

Similarly, the non-zero partials for the last term of Eq. (3–10) are given by:

$$\frac{\partial \mathbf{r}_M^B}{\partial \Delta \mathbf{E}_M} = \frac{\mu}{1 + \mu} \frac{\partial \mathbf{r}_M^E}{\partial \Delta \mathbf{E}_M} \quad (3-15)$$

$$\frac{\partial \mathbf{r}_M^B}{\partial \mu_E} = \frac{\mathbf{r}_M^E}{(1 + \mu)^2 \mu_M} \quad (3-16)$$

$$\frac{\partial \mathbf{r}_M^B}{\partial \mu_M} = - \frac{\mu \mathbf{r}_M^E}{(1 + \mu)^2 \mu_M} \quad (3-17)$$

The partial derivative of the Solar-System barycentric position vector of a planet (P) other than the Earth-Moon barycenter is given by:

$$\frac{\partial \mathbf{r}_P^C}{\partial \mathbf{q}} = \frac{\partial \mathbf{r}_P^S}{\partial \Delta \mathbf{E}_P} \quad (3-18)$$

which is interpolated from the planetary partials file. The partial derivative of the Solar-System barycentric position vector of the Sun with respect to reference parameters is given by:

$$\frac{\partial \mathbf{r}_S^C}{\partial \mathbf{q}} = 0 \quad (3-19)$$

except for the partial with respect to the *AU* scaling factor which is given by Eq. (3–8).

The partial derivative of the Solar-System barycentric position vector of an asteroid or a comet (P) is given by:

$$\frac{\partial \mathbf{r}_P^C}{\partial \mathbf{q}} = \frac{\partial \mathbf{r}_P^S}{\partial \mathbf{q}} \quad (3-20)$$

where the parameter vector \mathbf{q} includes Set III orbital element corrections (or the alternate Keplerian orbital parameters) and the cometary nongravitational parameters A_1 and A_2 . These partial derivatives are interpolated from the small-body partials file.

3.1.4 CORRECTING THE PLANETARY EPHEMERIS

The ODP user can estimate Set III corrections and then use program EPHCOR (ephemeris correction) to linearly differentially correct the Chebyshev polynomial coefficients on the planetary ephemeris. The ODP can be executed with the original planetary ephemeris or a differentially corrected one. The program cannot obtain an iterative solution for Set III corrections. It only estimates linear differential corrections for the planetary ephemeris being used.

3.2 SATELLITE EPHEMERIDES

3.2.1 DESCRIPTION

Interpolation of the satellite ephemeris for a planetary system produces the position (P), velocity (V), and acceleration (A) vectors of the satellites and the planet relative to the barycenter of the planetary system. These vectors have rectangular components referred to a space-fixed coordinate system which is nominally aligned with the mean Earth equator and equinox of J2000. It is assumed that each satellite ephemeris is aligned with the planetary ephemeris frame (PEF) of the particular planetary ephemeris used in executing the ODP. The time argument is seconds of coordinate time (ET) past J2000 in the Solar-System barycentric space-time frame of reference.

The satellite ephemerides were obtained from theories or from numerical integration. The process of forming a satellite ephemeris by numerical integration is described in Peters (1981). Jacobson (1997) describes the sources of the satellite ephemerides for Mars, Jupiter, Saturn, Uranus, Neptune, and Pluto. Although the source of each satellite ephemeris is different, the format of each working satellite ephemeris is the same. Each of the three components of the position of each satellite and the planet relative to the system barycenter is

SECTION 3

represented by an N th-degree expansion in Chebyshev polynomials. This representation is the same as that of the planetary ephemeris. Each of the three components of the velocity of each of these bodies is represented by an independent expansion in Chebyshev polynomials. The velocity components are only the same as differentiated position components if the ephemeris was generated by numerical integration. Acceleration components are obtained by replacing the Chebyshev polynomials in the N th-degree expansions in Chebyshev polynomials (for position components) with their second time derivatives.

3.2.2 POSITION, VELOCITY, AND ACCELERATION VECTORS INTERPOLATED FROM SATELLITE EPHEMERIDES

3.2.2.1 Interpolation of Satellite Ephemerides

For each satellite ephemeris, the interpolated position, velocity, and acceleration vectors of the satellites and the planet relative to the barycenter of the planetary system are in units of kilometers, kilometers per second, and kilometers per second squared, respectively.

Each satellite ephemeris contains the gravitational constant μ of the planetary system (*e.g.*, μ_J of the Jupiter system) in kilometers cubed per second squared. In the ODP, this system gravitational constant overstores the value obtained from the planetary ephemeris. Each satellite ephemeris also contains the gravitational constants of each planetary satellite in kilometers cubed per second squared. The system μ and the μ for each satellite can be estimated. The gravitational constant for the planet must be calculated as the system μ minus the sum of the gravitational constants of the satellites.

Each satellite ephemeris contains the position vector of the planet (0) relative to the barycenter (P) of the planetary system. However, it is more accurate to calculate it from the position vectors of the n satellites and the gravitational constants of the satellites and the planetary system:

$$\mathbf{r}_0^P = -\frac{1}{\mu_0} \sum_{i=1}^n \mu_i \mathbf{r}_i^P \quad \mathbf{r} \rightarrow \dot{\mathbf{r}}, \ddot{\mathbf{r}} \quad (3-21)$$

where μ_i is the gravitational constant of satellite i . The gravitational constant μ_0 of the planet is calculated from:

$$\mu_0 = \mu_P - \sum_{i=1}^n \mu_i \quad (3-22)$$

where μ_P is the gravitational constant of the planetary system.

3.2.2.2 Vectors Interpolated From Satellite Ephemerides

Satellite ephemerides are used in program Regres of the ODP to calculate the gravitational potential at the spacecraft, which is used to calculate the change in the time difference $ET - TAI$ (see Section 2) at the spacecraft during the transmission interval for one-way doppler (F_1) observables and one-way narrowband (*INS*) and wideband (*IWS*) spacecraft interferometry observables (see Section 11). For F_1 or one-way *IWS*, there are two one-way spacecraft light-time solutions. For one-way *INS*, there are four. For each of these light-time solutions, if the spacecraft is within the sphere of influence of one of the planetary systems Mars through Pluto, the satellite ephemeris for this planetary system is interpolated at the transmission time t_2 for the position, velocity, and acceleration vectors of the satellites and the planet. As noted above, the vectors for the planet are calculated from the vectors for the satellites using Eqs. (3-21) and (3-22).

If a landed spacecraft is resting upon a satellite or the planet of one of the outer planet systems, or the center of integration for the spacecraft ephemeris is one of these bodies, the satellite ephemeris for this planetary system must be interpolated at the transmission or reflection time t_2 for the position, velocity, and acceleration vectors of the body that the spacecraft is resting upon or the body that is the center of integration for the spacecraft ephemeris. These vectors are relative to the barycenter of the planetary system. If the body is the planet,

SECTION 3

use Eqs. (3–21) and (3–22). Furthermore, if the data type is F_1 or one-way *INS* or *IWS*, we need the position, velocity, and acceleration vectors of all of the satellites and the planet to calculate the gravitational potential at the lander or free spacecraft.

3.2.3 PARTIAL DERIVATIVES OF POSITION VECTORS INTERPOLATED FROM SATELLITE EPHEMERIDES

A satellite partials file for a planetary system contains the partial derivatives of the space-fixed position vectors of the satellites relative to the barycenter of the planetary system with respect to the solve-for parameters (\mathbf{q}). The rectangular components of these partial derivatives are represented by expansions in Chebyshev polynomials. This representation is the same as that used for position components on the planetary ephemeris. The partial derivative of the position vector of the planet relative to the barycenter of the planetary system with respect to the solve-for parameters is obtained (below) from the satellite partials by differentiating Eqs. (3–21) and (3–22) with respect to \mathbf{q} . Note that additional terms are obtained by differentiating the coefficients in these equations with respect to the gravitational constants of the satellites and the planetary system. If the satellite ephemeris was obtained from a theory, the parameter vector \mathbf{q} consists of the adjustable parameters of the theory. If the satellite ephemeris was obtained by numerical integration, \mathbf{q} consists of the state vectors (position and velocity components) of each satellite, the gravitational constants of each satellite and the planetary system, the right ascension and declination of the planet's pole and their time derivatives, and the zonal harmonic coefficients of the planet.

From Eq. (3–21), the partial derivative of the position vector of the planet (0) relative to the barycenter of the planetary system (P) due only to the variation of the satellite position vectors with \mathbf{q} is given by:

$$\frac{\partial \mathbf{r}_0^P}{\partial \mathbf{q}} = - \frac{1}{\mu_0} \sum_{i=1}^n \mu_i \frac{\partial \mathbf{r}_i^P}{\partial \mathbf{q}} \quad (3-23)$$

where the partial derivatives of the satellite position vectors with respect to \mathbf{q} are interpolated from the satellite partials file. The partial derivative of the position vector of the planet with respect to the gravitational constant μ_i of satellite i must be incremented by (obtained by differentiating the coefficients in Eqs. 3-21 and 3-22):

$$\frac{\partial \mathbf{r}_0^P}{\partial \mu_i} = -\frac{1}{\mu_0} [\mathbf{r}_i^P - \mathbf{r}_0^P] \quad (3-24)$$

The partial derivative of the position vector of the planet with respect to the gravitational constant μ_P of the planetary system must be incremented by:

$$\frac{\partial \mathbf{r}_0^P}{\partial \mu_P} = -\frac{1}{\mu_0} \mathbf{r}_0^P \quad (3-25)$$