SECTION 2

TIME SCALES AND TIME DIFFERENCES

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Figures

2.1 INTRODUCTION

This section is presented first because time is discussed in all of the other sections of this report. The various time scales used in programs PV and Regres of the ODP are described in Section 2.2. A time difference is the difference between values of an epoch recorded in two different time scales. Section 2.3 describes the time differences and gives the equations used for calculating them. Some of the time differences are obtained by interpolation of input files, which are described in Section 2.4. Section 2.5 presents time transformation trees. These figures indicate how to transform an epoch in one time scale to the corresponding epoch in any other time scale by adding and/or subtracting the intervening time differences. Time transformation trees are given for reception or transmission at a tracking station on Earth and at an Earth satellite.

Time in any time scale is represented as seconds past January 1, 2000, 12^{h} in that time scale. This epoch is J2000.0, which is the start of the Julian year 2000. The Julian Date for this epoch is JD 245,1545.0.

2.2 TIME SCALES

2.2.1 EPHEMERIS TIME (ET)

Ephemeris time (ET) means coordinate time, which is the time coordinate of general relativity. It is either coordinate time of the Solar-System barycentric space-time frame of reference or coordinate time of the local geocentric space-time frame of reference, depending upon which reference frame the ODP user has selected. It is the independent variable for the motion of celestial bodies, spacecraft, and light rays. The scale of ET in each of these two reference frames is defined below in Section 2.3.1.

2.2.2 INTERNATIONAL ATOMIC TIME (TAI)

International Atomic Time (TAI) is based upon the SI second (International System of Units). From p. 40–41 of the *Explanatory Supplement to* *the Astronomical Almanac* (1992), it is defined to be the duration of 9,192,631,770 periods of the radiation corresponding to the transition between two hyperfine levels of the ground state of the cesium-133 atom. It is further stated that this definition applies on the geoid (mean sea level). TAI is obtained from a worldwide system of synchronized atomic clocks. It is calculated as a weighted average of times obtained from the individual clocks, and corrections are applied for known effects.

Time obtained from a clock on board an Earth satellite will be referenced to satellite International Atomic Time. Satellite TAI is an imaginary time scale obtained from an ideal atomic clock on the satellite. It agrees on average with TAI obtained from atomic clocks on Earth.

2.2.3 UNIVERSAL TIME (UT1 AND UT1R)

Universal Time (UT) is the measure of time that is the basis for all civil time-keeping. It is an observed time scale, and the specific version used in the ODP is UT1. It is used to calculate mean sidereal time, which is the Greenwich hour angle of the mean equinox of date, measured in the true equator of date. Adding the equation of the equinoxes gives true sidereal time, which is used to calculate the position of the tracking station relative to the true equator and equinox of date. The equation for calculating mean sidereal time from observed UT1 is given in Section 5.3.6. From p. 51 of the *Explanatory Supplement to the Astronomical Almanac* (1992), the rate of UT1 is chosen so that a day of 86400 s of UT1 is close to the duration of the mean solar day. The phase of UT1 is chosen so that the Sun crosses the Greenwich meridian at approximately 12^h UT1.

Observed UT1 contains 41 short-period terms with periods between 5 and 35 days which are caused by long-period solid Earth tides. The algorithm for calculating the sum ΔUT1 of the 41 short-period terms of UT1 is given in Section 5.3.3. If ΔUT1 is subtracted from UT1, the result is called UT1R (where R means regularized). If UT1R is input to the ODP, the sum ΔUT1 must be calculated and added to UT1R to produce UT1, which is used to calculate mean sidereal time.

2.2.4 COORDINATED UNIVERSAL TIME (UTC)

Coordinated Universal Time (UTC) is standard time for 0° longitude. Since January 1, 1972, UTC uses the SI second and has been behind International Atomic Time TAI by an integer number of seconds. UTC is maintained within 0.90 s of observed UT1 by adding a positive or negative leap second to UTC. A leap second is usually positive, which has the effect of retarding UTC by one second; it is usually added at the end of June or December. After a positive leap second was added at the end of December, 1998, TAI − UTC increased from 31 s to 32 s; at the beginning of 1972, it was 10 s. The history of TAI − UTC is given in International Earth Rotation Service (1998), Table II-3, p. II-7.

2.2.5 GPS OR TOPEX MASTER TIME (GPS OR TPX)

GPS master time (GPS) is an atomic time scale, which is used instead of UTC as a reference time scale for GPS receiving stations on Earth and for GPS satellites. Similarly, TOPEX master time (TPX) is an atomic time scale used as a reference time scale on the TOPEX satellite. GPS time and TPX time are each an integer number of seconds behind TAI or satellite TAI. As opposed to UTC, these atomic time scales do not contain leap seconds. Therefore, the constant offsets from TAI or satellite TAI do not change.

2.2.6 STATION TIME (ST)

Station time (ST) is atomic time at a Deep Space Network (DSN) tracking station on Earth, a GPS receiving station on Earth, a GPS satellite, or the TOPEX satellite. These atomic time scales depart by small amounts from the corresponding reference time scales. The reference time scale for a DSN tracking station on Earth is UTC. For a GPS receiving station on Earth or a GPS satellite, the reference time scale is GPS master time (GPS). For the TOPEX satellite, the reference time scale is TOPEX master time (TPX). Note, the TPX and GPS time scales can be used for any Earth-orbiting spacecraft.

2.3 TIME DIFFERENCES

2.3.1 ET – TAI

2.3.1.1 The Metric Tensor and the Metric

This section gives the equations for the n-body metric tensor and the corresponding expression for the interval ds. All of the relativistic equations in programs PV and Regres of the ODP can be derived from these equations or from simplifications of them. The components of the Parameterized Post–Newtonian (PPN) n-body point-mass metric tensor, which contains the PPN parameters β and γ of Will and Nordtvedt (1972), are given by the following equations, where the subscripts 1 through 4 refer to the four space-time coordinates. Subscripts 1, 2, and 3 refer to position coordinates, and 4 refers to coordinate time t multiplied by the speed of light c.

$$g_{11} = g_{22} = g_{33} = -\left(1 + \frac{2\gamma}{c^2}\sum_{j \neq i}\frac{\mu_j}{r_{ij}}\right) \tag{2–1}$$

$$g_{pq} = 0 \quad (p, q = 1, 2, 3; p \neq q) \tag{2–2}$$

$$g_{14} = g_{41} = \frac{2 + 2\gamma}{c^3}\sum_{j \neq i}\frac{\mu_j \dot{x}_j}{r_{ij}} \tag{2–3}$$

$$g_{24} = g_{42} = \frac{2 + 2\gamma}{c^3}\sum_{j \neq i}\frac{\mu_j \dot{y}_j}{r_{ij}} \tag{2–4}$$

$$g_{34} = g_{43} = \frac{2 + 2\gamma}{c^3}\sum_{j \neq i}\frac{\mu_j \dot{z}_j}{r_{ij}} \tag{2–5}$$

$$g_{44} = 1 - \frac{2}{c^2}\sum_{j \neq i}\frac{\mu_j}{r_{ij}} + \frac{2\beta}{c^4}\left[\sum_{j \neq i}\frac{\mu_j}{r_{ij}}\right]^2 - \frac{1+2\gamma}{c^4}\sum_{j \neq i}\frac{\mu_j \dot{s}_j{}^2}{r_{ij}}$$
$$+ \frac{2(2\beta - 1)}{c^4}\sum_{j \neq i}\frac{\mu_j}{r_{ij}}\sum_{k \neq j}\frac{\mu_k}{r_{jk}} - \frac{1}{c^4}\sum_{j \neq i}\mu_j \frac{\partial^2 r_{ij}}{\partial t^2} \qquad (2\text{–}6)$$

where the indices j and k refer to the n bodies and k includes body i, whose motion is desired. Also,

μ_j	=	gravitational constant for body j.
	=	Gm_j, where G is the universal gravitational constant and m_j is the rest mass of body j.
c	=	speed of light.

Let the position, velocity, and acceleration vectors of body j, with rectangular components referred to a non-rotating frame of reference whose origin is located at the barycenter of the system of n bodies, be denoted by :

$$\mathbf{r}_j = \begin{bmatrix} x_j \\ y_j \\ z_j \end{bmatrix}; \qquad \dot{\mathbf{r}}_j = \begin{bmatrix} \dot{x}_j \\ \dot{y}_j \\ \dot{z}_j \end{bmatrix}; \qquad \ddot{\mathbf{r}}_j = \begin{bmatrix} \ddot{x}_j \\ \ddot{y}_j \\ \ddot{z}_j \end{bmatrix} \qquad (2\text{–}7)$$

where the dots denote differentiation with respect to coordinate time t. Then, r_{ij} and $\dot{s}_j^2$ can be obtained from:

$$r_{ij}{}^2 = (\mathbf{r}_j - \mathbf{r}_i) \cdot (\mathbf{r}_j - \mathbf{r}_i) \qquad (2\text{–}8)$$

$$\dot{s}_j{}^2 = \dot{\mathbf{r}}_j \cdot \dot{\mathbf{r}}_j \qquad (2\text{–}9)$$

From Eq. (2–8), the first and second partial derivatives of r_{ij} with respect to coordinate time t (obtained by holding the rectangular components of the position vector of body i fixed) are given by:

$$\frac{\partial r_{ij}}{\partial t} = \frac{\left(\mathbf{r}_j - \mathbf{r}_i\right) \cdot \dot{\mathbf{r}}_j}{r_{ij}} \tag{2–10}$$

$$\frac{\partial^2 r_{ij}}{\partial t^2} = \frac{\left(\mathbf{r}_j - \mathbf{r}_i\right) \cdot \ddot{\mathbf{r}}_j}{r_{ij}} + \frac{\dot{s}_j^{\,2}}{r_{ij}} - \frac{\left[\left(\mathbf{r}_j - \mathbf{r}_i\right) \cdot \dot{\mathbf{r}}_j\right]^2}{r_{ij}^{\;3}} \tag{2–11}$$

Since this equation is used to evaluate the last term of Eq. (2–6) which is of order $1/c^4$, and higher order terms are ignored, the acceleration of body j can be evaluated from Newtonian theory:

$$\ddot{\mathbf{r}}_j = \sum_{k \neq j} \frac{\mu_k \left(\mathbf{r}_k - \mathbf{r}_j\right)}{r_{jk}^{\;3}} \tag{2–12}$$

where k includes body i whose motion is desired.

The invariant interval ds between two events with differences in their space and time coordinates of dx^1, dx^2, dx^3, and dx^4 is given by

$$ds^2 = g_{pq}\, dx^p\, dx^q \tag{2–13}$$

where the repeated indices are summed over the integers 1 through 4 and g_{pq} is the n-body metric tensor given by Eqs. (2–1) to (2–6) and related equations. The four space-time coordinates are the three position coordinates of point i (where the interval ds is recorded) and the speed of light c multiplied by coordinate time t:

$$\left.\begin{aligned} x^1 &= x_i \\ x^2 &= y_i \\ x^3 &= z_i \\ x^4 &= ct \end{aligned}\right\} \tag{2–14}$$

Substituting the components of the metric tensor and the differentials of (2–14) into (2–13) gives

$$\begin{aligned} ds^2 = {} & g_{44}c^2dt^2 + g_{11}\left(dx_i^2 + dy_i^2 + dz_i^2\right) \\ & + 2g_{14}dx_i cdt + 2g_{24}dy_i cdt + 2g_{34}dz_i cdt \end{aligned} \tag{2–15}$$

All of the terms of this equation are required in order to calculate the n-body point-mass relativistic perturbative acceleration in the Solar-System barycentric frame of reference (Section 4.4.1). However, all other relativistic terms in programs PV and Regres of the ODP can be derived from Eq. (2–15), where each component of the metric tensor contains terms to order $1/c^2$ only. Substituting terms to order $1/c^2$ from Eqs. (2–1) to (2–6) into Eq. (2–15) and scaling the four space-time coordinates by the constant scale factor l gives

$$ds^2 = l^2\left[\left(1 - \frac{2U}{c^2}\right)c^2dt^2 - \left(1 + \frac{2\gamma U}{c^2}\right)\left(dx^2 + dy^2 + dz^2\right)\right] \tag{2–16}$$

where the subscript i has been deleted from the position components of point i, and $U > 0$ is the gravitational potential at point i which is given by

$$U = \sum_{j \neq i} \frac{\mu_j}{r_{ij}} \tag{2–17}$$

where the summation includes the bodies of the Solar System in the Solar-System barycentric frame of reference. In the local geocentric frame of reference, U is the gravitational potential due to the Earth only. The scale factor l, whose value is very close to unity, will be represented by

$$l = 1 + L \tag{2–18}$$

The scale factor l does not affect the equations of motion for bodies or light. However, it does affect the rate of an atomic clock, which records the interval ds divided by the speed of light c. The definitions for L which apply for the Solar-

System barycentric frame of reference and for the local geocentric frame of reference are defined below in Sections 2.3.1.2 and 2.3.1.3. Numerical values for L in these two frames of reference are not required in this section in order to obtain the various expressions for ET − TAI. However, they are required in Section 4.3 to transform the geocentric space-fixed position vector of the tracking station from the local geocentric frame of reference to the Solar-System barycentric frame of reference. They are also used in that section to transform the gravitational constant of the Earth from its value in the Solar-System barycentric frame of reference to its value in the local geocentric frame of reference.

An approximate solution to Einstein's field equations for the case of a massless particle moving in the gravitational field of n massive bodies was first obtained by Droste (1916). de Sitter (1915–1916 and 1916–1917) extended the work of Droste to account for the mass of the body whose motion is desired. However, he made a theoretical error in the calculation of one of his terms, which was corrected by Eddington and Clark (1938). The Droste/de Sitter/Eddington and Clark metric tensor is the same as Eqs. (2–1) to (2–6) and Eq. (2–11), if the PPN parameters β and γ are set to their general relativistic values of unity. The PPN metric of Will and Nordtvedt (1972) has a different form. However, Shahid-Saless and Ashby (1988) used a gauge transformation to transform the PPN metric to the Eddington and Clark metric. The resulting metric tensor given by Eqs. (11) to (13) of Shahid-Saless and Ashby (1988), with the PPN parameters ζ_1 and ζ_2 set to their general relativistic values of zero, is equal to (the negative of) the metric tensor given by Eqs. (2–1) to (2–6) and (2–11) above. The corresponding n-body Lagrangian was first derived by Estabrook (1971). The n-body point-mass relativistic perturbative acceleration given in Section 4.4.1 can be derived from the n-body metric tensor or the corresponding Lagrangian.

2.3.1.2 Solar-System Barycentric Frame of Reference

This section presents two expressions for coordinate time ET in the Solar-System barycentric frame of reference minus International Atomic Time TAI. In the expression given in Subsection 2.3.1.2.1, TAI is obtained from a fixed atomic clock at a tracking station on Earth. In the expression given in Subsection 2.3.1.2.2, TAI is obtained from an atomic clock on an Earth satellite. As stated above in Section 2.2.2, satellite TAI agrees on average with TAI obtained from fixed atomic clocks on Earth. An approximation for either of these two expressions for ET – TAI is given in Subsection 2.3.1.2.3.

In both expressions for ET – TAI, coordinate time ET and International Atomic Time TAI run on average at the same rate. Both of these expressions contain the same constant offset in seconds plus periodic terms. The specific coordinate time (ET) used in the ODP is referred to as Barycentric Dynamical Time (TDB) on p. 42 of the *Explanatory Supplement* (1992). From p. 41 of this reference, TDB shall differ from TAI + 32.184 seconds (exactly) by periodic terms only. Hence, the constant offset appearing in the expressions for ET – TAI will be 32.184 s. The *Explanatory Supplement* (1992) also refers (on p. 46) to Barycentric Coordinate Time (TCB) which differs from TDB in rate. This alternate form of coordinate time (TCB) is not used in the ODP.

The differential equation relating coordinate time ET in the Solar-System barycentric frame of reference and International Atomic Time TAI at a tracking station on Earth or on an Earth satellite can be obtained from Eq. (2–16). Since the differential equation and the resulting expression for ET – TAI will contain terms to order $1/c^2$ only, the second factor containing the gravitational potential U can be deleted. The resulting expression for the interval ds (which is called the metric) is the Newtonian approximation to the n-body metric.

An interval of proper time $d\tau$ recorded on an atomic clock is related to the interval ds along its world line by

$$d\tau = \frac{ds}{c} \tag{2–19}$$

Proper time τ will refer specifically to International Atomic Time TAI. In Eq. (2–16), t will refer specifically to coordinate time (ET) in the Solar-System barycentric frame of reference. In Eq. (2–18), it will be seen that the constant L is of order $1/c^2$. Substituting Eqs. (2–19) and (2–18) into (2–16), expanding and retaining terms to order $1/c^2$ gives the differential equation relating TAI and ET:

$$\frac{d\tau}{dt} = 1 - \frac{U}{c^2} - \frac{1}{2}\frac{v^2}{c^2} + L \tag{2–20}$$

where U is the gravitational potential (2–17) at the tracking station on Earth or at the Earth satellite, and v is the Solar-System barycentric velocity of the tracking station on Earth or the Earth satellite, given by

$$v^2 = \left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 + \left(\frac{dz}{dt}\right)^2 \tag{2–21}$$

From Eq. (2–20), TAI will run on average at the same rate as ET if the constant L has the value

$$L = \frac{1}{c^2}\langle U + \frac{1}{2}v^2 \rangle \tag{2–22}$$

where the brackets $\langle \quad \rangle$ denote the long-term time average value of the quantity contained within them. From (2–20) and (2–22), it can be seen that the desired expression for ET – TAI at a tracking station on Earth or at an Earth satellite can be obtained by integrating periodic variations in the gravitational potential U at this point and periodic variations in the square of the Solar-System barycentric velocity of this point.

The value of the constant L, which applies in the Solar-System barycentric frame of reference, is obtained in Section 4.3.1.2 by evaluating Eq. (2–22) at mean sea level on Earth. If L were evaluated at the location of an Earth satellite, a different value would be obtained. This offset value of L is used in Eq. (2–20) in order to force satellite TAI to run on average at the same rate as coordinate time ET in the Solar-System barycentric frame of reference. Any departure in the rate of atomic time on the Earth satellite from the rate of satellite TAI can be absorbed into the quadratic time offset described below in Section 2.3.5.

2.3.1.2.1 Tracking Station on Earth

Eq. (2–20) was evaluated in Moyer (1981) for proper time τ equal to International Atomic Time TAI obtained from an atomic clock located at a fixed tracking station on Earth. This equation was integrated to give an expression for coordinate time ET in the Solar-System barycentric frame of reference minus TAI obtained at a fixed tracking station on Earth. The derivation was simplified by using a first-order expansion of the gravitational potential and integration by parts. This technique was first applied to this problem by Thomas (1975). Moyer (1981) gives two different expressions for calculating ET − TAI at a tracking station on Earth. Eq. (46) of Part 1 is the "vector form" of the expression. It is a function of position and velocity vectors of various celestial bodies of the Solar System and the geocentric space-fixed position vector of the tracking station on Earth. This equation was converted to a function of time given by Eq. (38) of Part 2 and related equations. The ODP previously calculated ET − TAI as a function of time. However, it currently calculates ET − TAI from the vector form of the equation. The vector form is more accurate and easier to calculate. Furthermore, it was easier to modify the derivation of the vector form so that the resulting expression for ET − TAI applied for TAI obtained at an Earth satellite. However, evaluation of ET − TAI from the vector form of the equation sometimes requires the use of an iterative procedure because the required vectors are not always available until after the time difference is calculated.

Appendix A of Moyer (1981) describes the calculation of the computed values of two-way (same transmitting and receiving station) and three-way (different transmitting and receiving stations) range and doppler observables and shows how the ET − TAI time differences are used in these calculations. It also gives equations for the direct and indirect effects of various types of terms of ET − TAI on the computed values of these observables. The indirect effects are due to the effects of ET − TAI on the reception time at the receiving station, the reflection time at the spacecraft, and the transmission time at the transmitting station. Changes in these epochs have an indirect effect on the computed observables. Appendix B of Moyer (1981) develops criteria for the retained terms of ET − TAI. The accuracy of two-way range observables of the DSN is currently about 1–2 m in the one-way range ρ from the tracking station to the spacecraft. It was desired to limit the direct effect of neglected terms of ET − TAI on ρ to an RSS error of 1–2 m at a range ρ of 10 Astronomical Units (AU). The RSS direct error in computed two-way range observables due to neglected terms of ET − TAI, expressed as the equivalent change in the one-way range ρ, is 0.13 m per AU or 1.3 m at 10 AU. The accuracy of two-way doppler observables of the DSN is about 0.4 x 10^{-5} m/s in the one-way range rate $\dot{\rho}$ under the very best of conditions. The RSS direct error in computed two-way doppler observables due to neglected terms of ET − TAI, expressed as the equivalent change in $\dot{\rho}$, is 0.4 x 10^{-6} m/s per AU or 0.4 x 10^{-5} m/s at 10 AU. The RSS value of neglected terms of ET − TAI is about 4.2 μs. For a range rate of 30 km/s, this produces an indirect error in ρ of 0.13 m. For a spacecraft in heliocentric cruise, the indirect error in $\dot{\rho}$ is negligible. However, for a spacecraft near Jupiter where the acceleration can be about 25 m/s^2, the indirect error in $\dot{\rho}$ can be up to 10^{-4} m/s. For a Jupiter flyby, estimation of the spacecraft state vector relative to Jupiter will eliminate a constant error in ET − TAI, and consequently, the indirect error in $\dot{\rho}$ will be reduced to less than 10^{-6} m/s. For a Jupiter orbiter, the indirect error can be reduced by estimating the spacecraft state and a time-varying clock offset at the tracking station.

The vector form of the expression for coordinate time ET in the Solar-System barycentric frame of reference minus International Atomic Time TAI obtained from an atomic clock at a tracking station on Earth is Eq. (46) of Part 1 of Moyer (1981):

$$\begin{aligned}
\mathrm{ET} - \mathrm{TAI} = {} & 32.184\ \mathrm{s} + \frac{2}{c^2}\left(\dot{\mathbf{r}}_\mathrm{B}^\mathrm{S} \cdot \mathbf{r}_\mathrm{B}^\mathrm{S}\right) + \frac{1}{c^2}\left(\dot{\mathbf{r}}_\mathrm{B}^\mathrm{C} \cdot \mathbf{r}_\mathrm{E}^\mathrm{B}\right) + \frac{1}{c^2}\left(\dot{\mathbf{r}}_\mathrm{E}^\mathrm{C} \cdot \mathbf{r}_\mathrm{A}^\mathrm{E}\right) \\
& + \frac{\mu_\mathrm{J}}{c^2\left(\mu_\mathrm{S} + \mu_\mathrm{J}\right)}\left(\dot{\mathbf{r}}_\mathrm{J}^\mathrm{S} \cdot \mathbf{r}_\mathrm{J}^\mathrm{S}\right) + \frac{\mu_\mathrm{Sa}}{c^2\left(\mu_\mathrm{S} + \mu_\mathrm{Sa}\right)}\left(\dot{\mathbf{r}}_\mathrm{Sa}^\mathrm{S} \cdot \mathbf{r}_\mathrm{Sa}^\mathrm{S}\right) \\
& + \frac{1}{c^2}\left(\dot{\mathbf{r}}_\mathrm{S}^\mathrm{C} \cdot \mathbf{r}_\mathrm{B}^\mathrm{S}\right)
\end{aligned} \tag{2–23}$$

where

$\mathbf{r}_i^j$ and $\dot{\mathbf{r}}_i^j$ = space-fixed position and velocity vectors of point i relative to point j, km and km/s. They are a function of coordinate time ET, and the time derivative is with respect to ET.

Superscript or subscript C = Solar-System barycenter, S = Sun, B = Earth-Moon barycenter, E = Earth, M = Moon, J = Jupiter, Sa = Saturn, and A = location of atomic clock on Earth which reads TAI.

μ_S, μ_J, μ_{Sa} = gravitational constants of the Sun, Jupiter, and Saturn, km^3/s^2.

c = speed of light, km/s.

All of the vectors in Eq. (2–23) except the geocentric space-fixed position vector of the tracking station on Earth can be interpolated from the planetary ephemeris or computed from these quantities as described in Section 3. Calculation of the geocentric space-fixed position vector of the tracking station is described in Section 5. Section 7 gives algorithms for computing ET – TAI at the reception time or transmission time at a tracking station on Earth or an Earth satellite.

Eq. (2–23) for ET – TAI contains the clock synchronization term (listed below in the next paragraph) which depends upon the location of the atomic clock which reads International Atomic Time TAI and five location-independent periodic terms. The sum of the location-independent terms can also be obtained by numerical integration as described in Fukushima (1995). There are several alternate expressions for ET – TAI which have greater accuracies than Eq. (2–23) and more than 100 additional periodic terms. Fairhead and Bretagnon (1990) give an expression containing 127 terms with a quoted accuracy of 100 ns. They also have an expression containing 750 terms with an accuracy of 1 ns. Hirayama *et al*. (1987) present an expression containing 131 periodic terms with a quoted accuracy of 5 ns. Fukushima (1995) developed an extended version of this expression containing 1637 terms. These expressions were fit to the numerically integrated periodic terms of Fukushima (1995) for the JPL planetary ephemeris

DE245 (obtained from E. M. Standish[1]). In fitting the more accurate expression of Fairhead and Bretagnon (1990) to the numerical terms, some analytical terms were deleted, and the coefficients of an empirical correction term were estimated. The numerical minus analytical residuals for this modified expression (containing 515 terms) were less than 3 ns. For the other four unmodified expressions, the residuals varied from –131 ns to +64 ns.

2.3.1.2.2 Earth Satellite

The derivation of Eq. (2–23) is given in Moyer (1981). This derivation has been modified so that it applies for coordinate time ET in the Solar-System barycentric frame of reference minus satellite International Atomic Time TAI obtained from an atomic clock on an Earth satellite. The resulting expression for ET – TAI_{SAT}, where the subscript indicates that TAI is satellite TAI, is Eq. (2–23) with one term changed plus one new periodic term. The term of (2–23), which is changed, is the third periodic term on the right hand side:

$$\frac{1}{c^2}\left(\dot{\mathbf{r}}_{\mathrm{E}}^{\mathrm{C}} \cdot \mathbf{r}_{\mathrm{A}}^{\mathrm{E}}\right)$$

In this term, the point A no longer refers to the location of the tracking station on Earth. For this application, it refers to the position of the Earth satellite. The new periodic term is P_{SAT}:

$$P_{\mathrm{SAT}} = \frac{2}{c^2}\left(\dot{\mathbf{r}}_{\mathrm{SAT}}^{\mathrm{E}} \cdot \mathbf{r}_{\mathrm{SAT}}^{\mathrm{E}}\right) \qquad (2\text{–}24)$$

where $\mathbf{r}_{\mathrm{SAT}}^{\mathrm{E}}$ and $\dot{\mathbf{r}}_{\mathrm{SAT}}^{\mathrm{E}}$ are the geocentric space-fixed position and velocity vectors of the Earth satellite interpolated from the satellite ephemeris as a function of coordinate time ET of the Solar-System barycentric frame of reference. Applying these two changes to Eq. (2–23) gives the desired expression for coordinate time ET in the Solar-System barycentric frame of reference minus satellite TAI obtained from an atomic clock on an Earth satellite:

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[1] Unofficial interim version, never released.

$$\mathrm{ET} - \mathrm{TAI}_{\mathrm{SAT}} = [\mathrm{ET} - \mathrm{TAI}]_{\mathrm{A=SAT}} + P_{\mathrm{SAT}} \tag{2–25}$$

where the first term on the right hand side means Eq. (2–23) evaluated with $\mathbf{r}_{\mathrm{A}}^{\mathrm{E}}$ equal to the geocentric space-fixed position vector of the Earth satellite, $\mathbf{r}_{\mathrm{SAT}}^{\mathrm{E}}$, and P_{SAT} is given by Eq. (2–24). Interpolation of the planetary ephemeris and the satellite ephemeris at the ET value of the epoch will give all of the vectors required to evaluate Eq. (2–25).

2.3.1.2.3 Approximate Expression

A number of algorithms require an approximate expression for coordinate time ET in the Solar-System barycentric frame of reference minus International Atomic Time TAI at a tracking station on Earth or an Earth satellite. The approximate expression consists of the first two terms on the right hand side of Eq. (2–23) converted to a function of time. The second of these two terms is the 1.6 ms annual term. The remaining periodic terms of (2–23) have amplitudes of 21 μs or less. The second term on the right hand side of Eqs. (37) and (38) of Part 2 of Moyer (1981) is the 1.6 ms annual term with an analytical expression and a numerical value for the amplitude, respectively. The amplitude of this term is proportional to the eccentricity e of the heliocentric orbit of the Earth-Moon barycenter, which is given by the quadratic on p. 98 of the *Explanatory Supplement* (1961). Changing the value of e from its J1975 value of 0.01672 to its J2000 value of 0.01671 changes the amplitude of the 1.6 ms term from 1.658 ms to 1.657 ms. Hence, the approximate expression for ET – TAI in seconds at a tracking station on Earth or an Earth satellite in the Solar-System barycentric frame of reference is given by

$$\mathrm{ET} - \mathrm{TAI} = 32.184 + 1.657 \times 10^{-3} \sin E \tag{2–26}$$

where the eccentric anomaly of the heliocentric orbit of the Earth-Moon barycenter is given approximately by Eq. (40) of Part 2 of Moyer (1981):

$$E = M + 0.01671 \sin M \tag{2–27}$$

The mean anomaly M of the heliocentric orbit of the Earth-Moon barycenter is given by (in radians):

$$M = 6.239{,}996 + 1.990{,}968{,}71 \times 10^{-7}\, t \tag{2–28}$$

where t is ET or TAI in seconds past J2000.0. This linear expression is tangent to the cubic given on p. 98 of the *Explanatory Supplement* (1961) at J2000.

2.3.1.3 Geocentric Frame of Reference

The expression for the interval ds in the local geocentric frame of reference is Eq. (2–16) with the gravitational potential U replaced by the term of (2–17) due to the Earth. This is the one-body metric of Schwarzschild expressed in isotropic coordinates and containing all terms in the metric tensor to order $1/c^2$.

This section presents two expressions for coordinate time ET in the local geocentric frame of reference minus International Atomic Time TAI. In the expression given in Subsection 2.3.1.3.1, TAI is obtained from a fixed atomic clock at a tracking station on Earth. In the expression given in Subsection 2.3.1.3.2, TAI is satellite TAI obtained from an atomic clock on an Earth satellite.

In both expressions for ET − TAI, coordinate time ET in the local geocentric frame of reference and International Atomic Time TAI or satellite TAI run on average at the same rate. Both of these expressions contain the same constant offset of 32.184 s. The specific coordinate time ET used in these expressions is referred to as Terrestrial Dynamical Time (TDT) or Terrestrial Time (TT) on pp. 42 and 47 of the *Explanatory Supplement* (1992). This reference also refers (on pp. 46–47) to Geocentric Coordinate Time (TCG), which differs from TT in rate. This alternate form of coordinate time (TCG) in the geocentric frame is not used in the ODP.

The differential equation relating International Atomic Time TAI at a tracking station on Earth or satellite TAI recorded on an atomic clock on an Earth satellite (both denoted by τ), and coordinate time ET in the local geocentric frame of reference (denoted as t) is given by Eq. (2–20), where the constant L (denoted as L_{GC} in the geocentric frame of reference) is given by Eq. (2–22), the gravitational potential U is replaced by the term of (2–17) due to the Earth, and v given by (2–21) is the geocentric velocity of the tracking station or the Earth satellite.

The value of the constant L_{GC} which applies in the local geocentric frame of reference is obtained in Section 4.3 by evaluating Eq. (2–22), as modified in the preceding paragraph, at mean sea level on Earth. If L_{GC} were evaluated at the location of an Earth satellite, a different value would be obtained. This offset value of L_{GC} is used in Eq. (2–20) in order to force satellite TAI to run on average at the same rate as coordinate time ET in the geocentric frame of reference. Any departure in the rate of atomic time on the Earth satellite from the rate of satellite TAI can be absorbed into the quadratic time offset described below in Section 2.3.5.

2.3.1.3.1 Tracking Station on Earth

For a fixed atomic clock at a tracking station on Earth, the gravitational potential at the clock due to the Earth and the geocentric velocity of the clock are nearly constant, and periodic variations in these quantities will be ignored. Hence, the constant values of U and v in (2–20) cancel the corresponding values in (2–22) and (2–20) reduces to

$$\frac{d\tau}{dt} = 1 \tag{2–29}$$

and coordinate time ET in the local geocentric frame of reference minus International Atomic Time TAI at a tracking station on Earth is a constant:

$$\mathrm{ET} - \mathrm{TAI} = 32.184 \text{ s} \tag{2–30}$$

From pp. 42 and 47 of the *Explanatory Supplement* (1992), Terrestrial Dynamical Time (TDT) or Terrestrial Time (TT), denoted here as coordinate time ET in the local geocentric frame of reference, minus International Atomic Time TAI is equal to 32.184 s.

2.3.1.3.2 Earth Satellite

For satellite International Atomic Time TAI obtained from an atomic clock on an Earth satellite which is moving on a geocentric elliptical orbit, the gravitational potential U at the satellite due to the Earth and the square of the geocentric velocity v of the satellite in Eq. (2–20) will vary periodically from their average values in (2–22) due to the eccentricity of the elliptical orbit. Using the point-mass gravitational potential due to the Earth, Eqs. (2–20) and (2–22) can be integrated to give the following expression for coordinate time ET in the local geocentric frame of reference minus satellite TAI:

$$\mathrm{ET} - \mathrm{TAI}_{\mathrm{SAT}} = 32.184 \text{ s} + P_{\mathrm{SAT}} \tag{2–31}$$

where P_{SAT} is given by Eq. (2–24). The geocentric space-fixed position and velocity vectors of the Earth satellite in (2–24) are interpolated from the satellite ephemeris at the ET value of the epoch. Note that the form of P_{SAT}, which is due to the elliptical orbit of the satellite about the Earth, is the same as the first periodic term of (2–23), which is due to the elliptical orbit of the Earth-Moon barycenter about the Sun. In each case, one-half of the term is due to the variation in the gravitational potential of the central body, and the other half of the term is due to the variation in the square of the velocity.

2.3.2 TAI – UTC

From Section 2.2.4, TAI – UTC is an integer number of seconds. Its value at any given time can be obtained by interpolating either of the input files for time differences discussed below in Section 2.4 with the UTC value of the epoch as the argument.

2.3.3 TAI – GPS AND TAI – TPX

From Section 2.2.5, TAI – GPS and TAI – TPX are constants. The user can input the values of these constants to the ODP on the General Input Program (GIN) file.

2.3.4 TAI – UT1 AND TAI – UT1R

Universal Time UT1 and its regularized form UT1R were discussed in Section 2.2.3. The value of TAI – UT1 or UT1R can be obtained by interpolating either of the input files for time differences as discussed in Section 2.4.

2.3.5 QUADRATIC OFFSETS BETWEEN STATION TIME ST AND UTC OR (GPS OR TPX) MASTER TIME

Section 2.2.6 discussed station time ST at a DSN tracking station on Earth, a GPS receiving station on Earth, a GPS satellite, and the TOPEX satellite. Each of these atomic time scales departs by a small amount from the corresponding reference time scale. The reference time scale is UTC for a DSN tracking station on Earth, GPS Master Time (GPS) for a GPS receiving station on Earth or a GPS satellite, and TOPEX Master Time (TPX) for the TOPEX satellite. The time differences UTC – ST, GPS – ST at a GPS receiving station on Earth or a GPS satellite, or TPX – ST are all represented by the following quadratic function of time:

$$(\text{UTC or GPS or TPX}) - \text{ST} = a + b\left(t - t_0\right) + c\left(t - t_0\right)^2 \qquad (2\text{–}32)$$

where a, b, and c are quadratic coefficients specified by time block with start time t_0 at each station or satellite, and t is the current time. The time scale for t and t_0 is either of the two time scales related by (2–32).

2.4 INPUT FILES FOR TIME DIFFERENCES, POLAR MOTION, AND NUTATION ANGLE CORRECTIONS

Some of the time differences used in the ODP are obtained by interpolation of either of two different input files that the ODP can read. The older of these two files is the so-called STOIC file (named after the program which originally created it) which contains the TP (timing and polar motion) array. This array contains the time differences TAI – UTC and TAI – UT1 or UT1R, the X and Y coordinates of the Earth's true pole of date relative to the mean pole of 1903.0 (defined in Section 5.2.5), and the time derivatives of each of these four quantities at each time argument, which is specifically UTC. The fixed size of the TP array limits the timespan of the data to about three years if the data is spaced a month apart. The newer of these two files is the Earth Orientation Parameter (EOP) file. It contains the four quantities which are in the TP array plus the corrections $\delta\psi$ and $\delta\varepsilon$ to the nutations in longitude $\Delta\psi$ and obliquity $\Delta\varepsilon$, respectively (defined in Section 5.3.5). The nominal values of the two nutation angles are obtained from the 1980 IAU (International Astronomical Union) Theory of Nutation (Seidelmann, 1982). The EOP file contains the values of these six quantities at each time argument, which is UTC. It does not contain the time derivatives of the six quantities. The file is open-ended and the data spacing is usually about a day.

For each quantity in the TP array, the value and rate at each of two successive time points defines a cubic. The cubic and its time derivative can be evaluated at the interpolation time. The only exception to this is TAI – UTC which is constant between two successive time points. Interpolation of each quantity on the EOP file, except TAI – UTC, requires the value of the quantity at each of four successive time points. The algorithm and code are due to X X Newhall. The first three points are fit to a quadratic, which is differentiated to give the derivative at the second point. Applying the same procedure to the last three points gives the derivative at point three. The values and derivatives at points two and three produce a cubic that is valid between these two points. The cubic and its time derivative can be evaluated at the interpolation time which must be between points two and three. Note that interpolation of each of these two files produces a continuous function and its derivative.

Interpolation of the TP array yields TAI – UT1 or UT1R, whichever is input. If it is the latter, program Regres calculates ΔUT1 (see Section 2.2.3) and subtracts it from TAI – UT1R to give TAI – UT1. If the EOP file contains

TAI – UT1, the interpolation program converts it internally to TAI – UT1R, which is the quantity that is always interpolated. The program calculates ΔUT1, which is subtracted from the interpolated quantity to give TAI – UT1, which is always the output quantity.

The quantities on the EOP file, Earth-fixed station coordinates (see Section 5), quasar coordinates (Section 8), and the frame-tie rotation matrix (Section 5) are determined on a real-time basis at the Jet Propulsion Laboratory (JPL) by fitting to Very Long Baseline Interferometry (VLBI) data, Lunar Laser Ranging (LLR) data, and data obtained from the International Earth Rotation Service (IERS). The data in the TP array currently comes from the same solution. Previously, it was obtained from the IERS.

2.5 TIME TRANSFORMATION TREES

This section presents two time transformation trees that show how the reception time in station time ST or the transmission time in coordinate time ET at a fixed tracking station on Earth or an Earth satellite is transformed to all of the other time scales. Each time transformation tree shows the route or path that must be taken to transform the ST or ET value of the epoch to the corresponding values in all of the other time scales. In general, each time transformation tree is not an algorithm which must be evaluated at a particular place in the code. Instead, each time transformation tree is broken into several parts, which are evaluated in different parts of the code. When the calculation of time transformations is described in the various sections of this report, the corresponding parts of the calculations described in the following five subsections will be referenced.

In the time transformation trees, ET refers to coordinate time in the Solar-System barycentric frame of reference or to coordinate time in the local geocentric frame of reference, depending upon which frame of reference has been specified by the ODP user.

The reception time in station time ST is the known data time tag for a range data point. For a doppler data point, it is the time tag for the data point plus or minus one-half of the count interval (see Section 13). For quasar VLBI data points, the reception time in station time ST at station 1 is the data time tag for wideband data. For narrowband data, it is the time tag plus or minus one-half of the count interval (see Section 13). The transmission time in coordinate time ET at a tracking station on Earth or an Earth satellite is obtained from the spacecraft light-time solution. The reception time in coordinate time ET at station 2 for a quasar VLBI data point is obtained from the quasar light-time solution.

2.5.1 RECEPTION AT DSN TRACKING STATION ON EARTH

Fig. 2–1 shows the time transformation tree used at the reception time or transmission time at a tracking station on Earth. For a DSN tracking station, Coordinated Universal Time UTC is used and GPS master time is not used. This section will evaluate the time transformations in Fig. 2–1 at the reception time t_3 at a DSN tracking station on Earth.

Calculate UTC – ST from Eq. (2–32) using t_3(ST) as the argument. Add UTC – ST to t_3(ST) to give t_3(UTC). Use it as the argument to interpolate the TP array or the EOP file for the value of TAI – UTC. Add it to t_3(UTC) to give t_3(TAI). Use it as the argument to calculate ET – TAI from Eq. (2–23) or (2–30) using the algorithm given in Section 7.3.1. Add ET – TAI to t_3(TAI) to give t_3(ET). The algorithm for computing ET – TAI also produces all of the position, velocity, and acceleration vectors required at t_3(ET).

One of these vectors is the geocentric space-fixed position vector of the tracking station, which is computed from the formulation of Section 5. In order to calculate this vector, the argument t_3(ET) must be transformed to t_3(UTC) and used as the argument to interpolate the TP array or the EOP file for TAI – UT1, the X and Y coordinates of the Earth's true pole of date, and, if the latter file is used, the nutation corrections $\delta\psi$ and $\delta\varepsilon$. The time difference TAI – UT1 is subtracted from t_3(TAI) to give t_3(UT1).

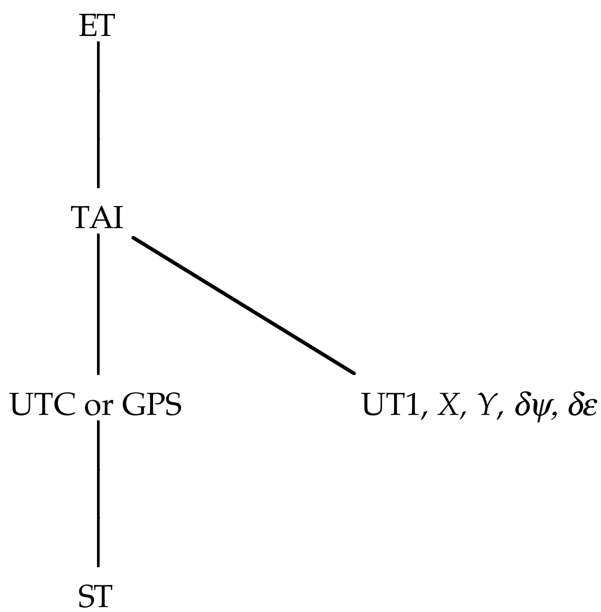

Figure 2–1 Time Transformations at a Tracking Station on Earth

The transformation of t_3(ET) to t_3(UTC) is accomplished as follows. Calculate ET – TAI from Eq. (2–23) in the Solar-System barycentric frame of reference or from (2–30) in the local geocentric frame of reference. In the former case, the geocentric space-fixed position vector of the tracking station is computed as a function of ET from the simplified algorithm given in Section 5.3.6.3. Subtract ET – TAI from t_3(ET) to give t_3(TAI). Use it as the argument to interpolate the TP array or the EOP file for TAI – UTC, and subtract it from t_3(TAI) to give t_3(UTC). Use it as the argument to re-interpolate the TP array or the EOP file for TAI – UTC and subtract it from t_3(TAI) to give the final value of t_3(UTC). Near a leap second in UTC, the second value obtained for UTC may differ from the first value by exactly one second.

2.5.2 RECEPTION AT GPS RECEIVING STATION ON EARTH

For a GPS receiving station on Earth, ST (see Fig. 2–1) is referred to GPS (GPS master time) and not to UTC. Calculate GPS – ST from Eq. (2–32) using t_3(ST) as the argument. Add GPS – ST to t_3(ST) to give t_3(GPS). Obtain TAI – GPS from the GIN file and add it to t_3(GPS) to give t_3(TAI). Use it as the argument to calculate ET – TAI from Eq. (2–23) or (2–30) using the algorithm given in Section 7.3.1. Add ET – TAI to t_3(TAI) to give t_3(ET). The algorithm for computing ET – TAI also produces all of the position, velocity, and acceleration vectors required at t_3(ET). The last two paragraphs of Section 2.5.1 also apply here.

2.5.3 RECEPTION AT THE TOPEX SATELLITE

Fig. 2–2 shows the time transformation tree used at the reception time or transmission time at an Earth satellite. For the TOPEX satellite, station time ST is referred to TPX (TOPEX master time). Calculate TPX – ST from Eq. (2–32) using t_3(ST) as the argument. Add TPX – ST to t_3(ST) to give t_3(TPX). Obtain TAI – TPX from the GIN file and add it to t_3(TPX) to give t_3(TAI). Use it as the argument to calculate ET – TAI from Eq. (2–25) or (2–31) using the algorithm given in Section 7.3.3. Add ET – TAI to t_3(TAI) to give t_3(ET).

The algorithm for computing ET – TAI also produces all of the position, velocity, and acceleration vectors required at t_3(ET).

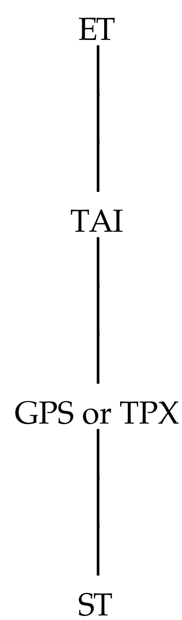

Figure 2–2 Time Transformations at an Earth Satellite

2.5.4 TRANSMISSION AT DSN TRACKING STATION ON EARTH

The time transformation tree shown in Fig. 2–1 is used at the transmission time t_1(ET) at a DSN tracking station on Earth. It is also used at the reception time t_2(ET) at station 2 on Earth for a quasar VLBI data point. This epoch, which will be denoted here as t_1(ET), and all of the required position, velocity, and acceleration vectors at this epoch are available from the spacecraft light-time solution (see Section 8.3) or the quasar light-time solution (Section 8.4). The geocentric space-fixed position vector of the tracking station is calculated in either of these two light-time solutions by using the time transformations described above in the last two paragraphs of Section 2.5.1.

Using t_1(ET) as the argument, calculate ET – TAI from Eq. (2–23) or (2–30). In the former equation, all of the required position and velocity vectors are available from the light-time solution. Subtract ET – TAI from t_1(ET) to give t_1(TAI). Using t_1(TAI) as the argument, interpolate the TP array or the EOP file for TAI – UTC and subtract it from t_1(TAI) to give t_1(UTC). Using it as the argument, re-interpolate the TP array or the EOP file for TAI – UTC and subtract it from t_1(TAI) to give the final value of t_1(UTC). Use it as the argument to calculate UTC – ST from Eq. (2–32), and subtract it from t_1(UTC) to give t_1(ST).

2.5.5 TRANSMISSION AT A GPS SATELLITE

The time transformation tree shown in Fig. 2–2 is used at the transmission time t_2(ET) at a GPS satellite. This epoch and all of the required position, velocity, and acceleration vectors at this epoch are available from the spacecraft (the GPS satellite) light-time solution (Section 8.3).

Using t_2(ET) as the argument, calculate ET – TAI from Eq. (2–25) or (2–31), where all of the required position and velocity vectors are available from the light-time solution. Subtract ET – TAI from t_2(ET) to give t_2(TAI). Obtain TAI – GPS from the GIN file and subtract it from t_2(TAI) to give t_2(GPS). Use it as the argument to calculate GPS – ST from Eq. (2–32), and subtract it from t_2(GPS) to give t_2(ST).