CHAPTER 4

TRANSFERS TO LOW LUNAR ORBITS

4.1 EXECUTIVE SUMMARY

This chapter examines low-energy transfers that target low, 100-kilometer (km), polar lunar orbits. The analyses presented here may be applied to any lunar orbit insertion; polar orbits are used as examples since mapping missions have historically been frequently sent to near-polar orbits about the Moon. This chapter presents surveys of direct transfers as well as low-energy transfers to low lunar orbits, and provides details about how to construct a desirable transfer, be it a short-duration direct transfer or a longer duration low-energy transfer.

Figure 4-1 shows an example direct transfer, compared with an example low-energy transfer to low lunar orbits. Much like the transfers presented in Chapter 3, these trajectories are ballistic in nature; they require a standard trans-lunar injection (TLI) maneuver, a few trajectory correction maneuvers, and an orbit insertion maneuver. One may again add Earth phasing orbits and/or lunar flybys to the trajectories, if needed, which change their performance characteristics.

Many thousands of direct and low-energy trajectories are surveyed in this chapter. Table 4-1 provides a quick guide for several types of transfers that are presented here, much like Table 3-1 from Chapter 3, comparing their launch energy costs, the breadth

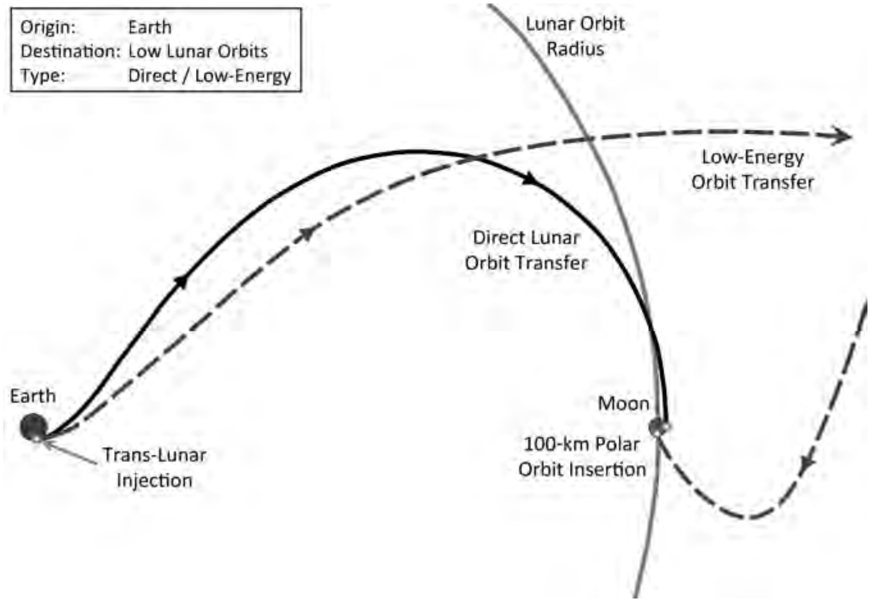

Figure 4-1 The profiles for both a direct and a low-energy transfer from the Earth to a low lunar orbit.

Table 4-1 A summary of several parameters that are typical for different mission scenarios to low lunar orbits. EPOs = Earth Phasing Orbits, BLT = Low-Energy Ballistic Lunar Transfer.

Mission Element	Direct Transfer	Direct w/EPOs	Simple BLT	BLT w/Outbound Lunar Flyby	BLT w/EPOs
Launch C_3 (km^2/s^2)	−2.2 to −1.5	< −1.5	−0.7 to −0.4	−2.1 to −0.7	< −1.5
Launch Period	Short	Extended	Extended	Short	Extended
Transfer Duration (days)	2–6	13+	70–120+	70–120+	80–130+
Outbound Lunar Flyby	No	No	No	Yes	Yes
Lunar Orbit Insertion ΔV (m/s)	∼820+	∼820+	∼640+	∼640+	∼640+

of their launch period, that is, the number of consecutive days they may be launched, their transfer duration, and the relative magnitude of the orbit insertion change in velocity (ΔV) upon arriving at the lunar orbit. The performance parameters are very similar to low-energy transfers to lunar libration orbits, except for the orbit insertion

ΔV. These parameters are representative and may be used for high-level mission design judgements, though the details will likely vary from mission to mission.

Conventional lunar mission design is presented in Section 4.3 as a reference for the analyses of low-energy lunar transfers. The trajectories shown in that section require trans-lunar injection parameter (C_3) values of at least -2.06 kilometers squared per second squared (km^2/s^2), realistic transfer durations between 2 and 6 days, and lunar orbit insertion ΔV values of at least 813 m/s. One can certainly construct quicker or longer transfers, but the injection C_3 and lunar orbit insertion ΔV values increase rapidly.

Direct transfers and low-energy transfers to low lunar orbits are directly compared and analyzed in Section 4.4. The surveys include many thousands of lunar transfers, arriving at the Moon in any orientation and arriving at different times. The surveys demonstrate that direct transfers must arrive at the Moon in a geometry such that the orbital plane is roughly normal to the Earth–Moon line at the time of arrival. Whereas low-energy transfers may be constructed that arrive at any orbital plane. If a mission must enter a lunar orbit with a particular node, then only certain values of the orbit's argument of periapse may be targeted, depending on the lunar arrival date; further, those values are different for low-energy transfers than they are for direct lunar transfers. It has been found that low-energy transfers require trans-lunar injection C_3 values of about $-0.6\ km^2/s^2$, compared with typical direct transfers that require C_3 values of about $-2.0\ km^2/s^2$. Low-energy transfers require about 70–120 days of transfer duration, compared with direct transfers that require 2–6 days, though either type of transfer may be designed to take more time. The lunar orbit insertion ΔV is at least 640 m/s for low-energy transfers, assuming an impulsive maneuver to immediately target a 100-km circular lunar orbit. Direct lunar transfers require at least 120 m/s more ΔV, and often significantly more ΔV than that to target the same arrival conditions. Finally, low-energy lunar transfers exist in families, such that very similar transfers exist to neighboring libration orbits. Very similar transfers also exist to the same orbit when the arrival time or arrival geometry is adjusted.

4.2 INTRODUCTION

This chapter is devoted to the analysis and construction of low-energy transfers to low lunar orbit. This is a rich problem; it is far too complex to present all possible examinations of such transfers in a concise form. To simplify the problem, while retaining a connection to practical spacecraft mission design, this book limits the scope of this study and only examines low-energy transfers to low-altitude, 100-km circular, polar orbits about the Moon. These orbits are very similar to many mapping orbits flown by historical lunar missions, including *Lunar Prospector* [56], *Kaguya/SELENE* [187], *Chang'e 1* [58], *Chandrayaan-1* (*CH-1*) [3], the *Lunar Reconnaissance Orbiter* (*LRO*) [188], and *Gravity Recovery and Interior Laboratory* (*GRAIL*) [83]. The procedures presented in this chapter may easily be applied to transfers that implement an eccentric capture orbit about the Moon: in that case the argument of periapse of the target orbit becomes a design constraint and the orbit insertion ΔV is reduced appropriately. This chapter contains all of the information to design such orbit insertions, assuming that the mission performs lunar orbit insertion (LOI) at an altitude of 100 km and an inclination of 90 degrees (deg). Even so, the procedures presented here may be applied to orbit insertions at other altitudes and in other inclinations, though in those cases the design space will have to be reconstructed by the mission designer. The surveys presented here provide a good representation of the trade space of any direct and low-energy transfer to any low orbit about the Moon.

Although the general characteristics of low-energy transfers to low lunar orbits are similar to the characteristics of low-energy transfers to lunar libration orbits, such as those presented in Chapter 3, the geometry of transfers that arrive at polar orbits is still significantly different. Therefore, the analysis in this chapter is independent of Chapter 3 and specifically tailored to study missions to low lunar orbit.

The *GRAIL* mission is the only mission in history, prior to 2012, to implement a low-energy transfer to a low-altitude orbit about the Moon as part of its primary mission. Its design features will be used as a reference in many of the discussions in this chapter [83–85]. *GRAIL*'s trajectory design is illustrated in Fig. 4-2, including the first and last launch opportunity in a 21-day launch period. This is the launch period published in Ref. 83; however, it was actually extended by many days as the mission developed. The *GRAIL* mission launched on September 10, 2011, on the third day of its launch period. *GRAIL*'s mission design includes two significant

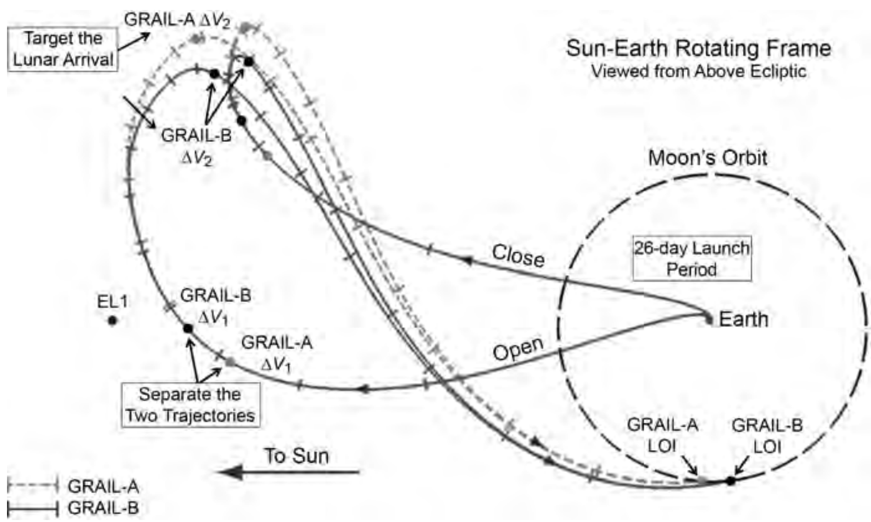

Figure 4-2 An illustration of *GRAIL*'s mission design, including a 26-day launch period and two deterministic maneuvers for both *GRAIL*-A and *GRAIL*-B, designed to separate their lunar orbit insertion times by 25 hours [83] (Originally published by the American Astronautical Society).

deterministic maneuvers performed per spacecraft during the cruise, performed primarily to separate their lunar orbit insertion dates. The trajectories generated in this chapter do not include these sorts of maneuvers. Chapter 6 explores the addition of maneuvers like those in *GRAIL*'s design.

4.3 DIRECT TRANSFERS BETWEEN EARTH AND LOW LUNAR ORBIT

The purpose of this book is to illustrate the costs, benefits, and characteristics of low-energy lunar transfers; the primary referent is the *direct* lunar transfer, which has been used so frequently in lunar missions that it is known as the conventional method. The first spacecraft launched toward the Moon, *Luna 1*, followed a direct transfer: a trajectory that required only 34 hours to reach the Moon, passing by within 6000 km of the surface. Since then, dozens of missions have implemented direct lunar transfers with durations ranging from 1.4 to 5.5 days, not including any staging orbits. Table 1-2 on page 16 summarize many example missions that implemented such direct transfers. Many resources exist that describe these direct lunar transfers in great detail [189]. This section only considers the ΔV of basic transfers as a function of the transfer duration to be used as a reference when describing low-energy lunar transfers.

Direct lunar transfers are trajectories that depend only on the gravity of the Earth and Moon. The Sun's gravity is accounted for, but only as a perturbation to the transfer. A very short-duration direct transfer departs the Earth on a hyperbola that encounters the Moon. The most efficient direct transfers typically require 4–5 days, depending on the location of the Moon in its elliptical orbit, and resemble Hohmann transfers. Figure 4-3 illustrates several direct lunar transfers that have varying transfer durations.

Figure 4-3 illustrates how the ΔV cost of a direct transfer increases away from the optimal transfer duration. But the cost doesn't rise very rapidly until the transfer duration has changed by several days. Recent spacecraft have taken advantage of the optimal transfer durations to maximize the amount of payload sent to the Moon. Conversely, it is apparent why the Apollo mission planners opted for a shorter transfer: the ΔV cost does not rise very much by decreasing the transfer duration from 4.5 days to 3.0 days, but the other consumables (including items such as food, water, and electrical power) required 1.5 days less support time on both the outbound and return transfer segments.

Since a spacecraft following a direct transfer only requires a few days to reach the Moon, it must be prepared to perform a maneuver within hours, or perhaps at most a day, to perform a trajectory correction maneuver. If this is an undesirably short amount of time, the mission may implement an Earth phasing orbit to extend the transfer duration. The spacecraft would be launched into an orbit that does not encounter the Moon, and only after one or more perigee passages would the trajectory finally arrive at the Moon.

The launch periods for many historical direct transfers were very short: only a handful of opportunities to launch per month, when the geometry was aligned

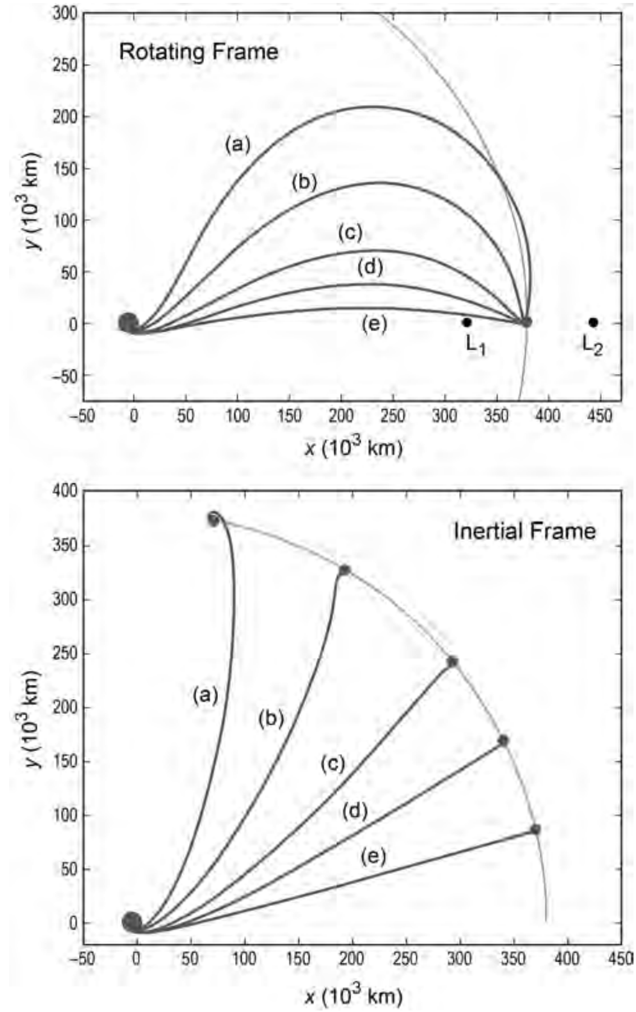

Figure 4-3 Five example direct transfers from 185-km circular Earth orbits to 100-km prograde lunar orbits, shown in the rotating frame (top) and inertial frame (bottom). These trajectories have been generated in the planar circular restricted three-body system. The following information applies to the labeled trajectories:

Traj.	Duration (days)	C_3 (km^2/s^2)	ΔV_{TLI} (km/s)	ΔV_{LOI} (km/s)	Total ΔV (km/s)
(a)	6.0	−1.976	3.138	0.829	3.966
(b)	4.5	−2.064	3.134	0.813	3.948
(c)	3.0	−1.670	3.152	0.893	4.045
(d)	2.0	0.264	3.240	1.248	4.488
(e)	1.0	13.654	3.831	3.024	6.854

properly. The *Clementine* and *Chandrayaan-1* missions implemented Earth phasing orbits, which extended the launch periods. *Chandrayaan-1*'s nominal mission profile included half a dozen Earth orbits prior to the lunar encounter. If the mission launched a day late, then the orbital period of one or more of these orbits would be adjusted to compensate for the change in transfer duration. The drawbacks of Earth phasing orbits include an extended operational timeline, which may add to the costs of the mission, and an increased dose of radiation as the spacecraft passes through the Van Allen Belts multiple times.

4.4 LOW-ENERGY TRANSFERS BETWEEN EARTH AND LOW LUNAR ORBIT

This section discusses how to build a low-energy ballistic transfer between the Earth and a low lunar orbit. The algorithms and methodology used to build a low-energy transfer are first described. Then, several example surveys are conducted, examining low-energy transfers that arrive at the Moon in some particular geometry at some given arrival time. The surveys become more general as this analysis continues. It then shows how to construct a map that tracks the minimum transfer ΔV cost required for a spacecraft to target any lunar orbit at a particular arrival time. Finally, the arrival time is opened up and transfers are examined that arrive at the Moon at many different times. The goal is to capture the transfer ΔV cost for transfers to any polar orbit about the Moon at any given arrival time in order to guide mission planners as they define the orbits and timeline for a given mission.

4.4.1 Methodology

Each transfer in the surveys presented here departs the Earth, coasts to the Moon, and injects directly into a low lunar orbit. To reduce the scope of the problem while still yielding practical data, the surveys presented here assume that the mission targets a circular 100-km polar orbit about the Moon. This lunar orbit is akin to the mapping orbits of several spacecraft, including *Lunar Prospector* [56], *Kaguya/SELENE* [187], *Chang'e 1* [58], *Chandrayaan-1* [3], the *LRO* [188], and *GRAIL* [83].

The LOI is modeled as a single impulsive maneuver that is performed at the periapse point and places the vehicle directly into a circular orbit. This is not a realistic maneuver, but it is useful to directly compare the total insertion cost of one transfer to another. The orbit insertion cost needed to place a satellite into an elliptical orbit, rather than a circular orbit, may be determined via the Vis-Viva equation [97].

The surveys presented here have been generated using a method that does not make many assumptions about what the lunar transfers look like. This permits each survey to reveal trajectories that may not have been expected. Each trajectory in each survey is constructed using the following procedure:

1. Construct the target lunar orbit. The following parameters are used in this study, specified in the International Astronomical Union (IAU) Moon Pole coordinate frame (see Section 2.4.4).

Periapse radius,	r_p :	1837.4 km (∼100-km altitude)
Eccentricity,	e :	ϵ
Equatorial inclination,	i :	90 deg
Argument of periapse,	ω :	Specified value
Longitude of the ascending node,	Ω :	Specified value
True anomaly,	ν :	0 deg

The argument of periapse is undefined for a circular orbit. However, since all practical missions to date have inserted into elliptical orbits, and some missions remain in a highly elliptical orbit, the target orbit's argument of periapse, ω, is presented here rather than the true anomaly, which is kept at 0 deg to indicate that LOI is performed at periapse. The orbit's eccentricity is given as ϵ: it is approximately zero (1×10^{-9}) while permitting ω to be defined. One may also use the argument of latitude, which is defined for a circular orbit.

2. Construct the LOI state.
   (a) Specify the date of the LOI, t_{LOI}. Dates are given here in Ephemeris Time (ET).
   (b) Specify the magnitude of the impulsive orbit insertion maneuver, ΔV_{LOI}. Apply the ΔV in a tangential fashion to the LOI state.
3. Propagate the state backward in time for 160 days.
4. Identify the perigee and perilune passages that exist in the trajectory.
   (a) If the trajectory flies by the Moon within 500 km, label the trajectory as undesirable.
   (b) The latest perigee passage that approaches within 500 km of the Earth is considered the earliest opportunity to inject into that trajectory.
   (c) If no low perigees are observed, then the lowest perigee is identified as the trans-lunar injection (TLI) location.
5. Characterize the performance of the trajectory, making note of the following values:
   - TLI altitude, inclination, and C_3;
   - Duration of the transfer;
   - Periapse altitude of any/all Earth and Moon flybys; and
   - LOI ΔV magnitude.

This procedure requires four inputs: the longitude of the ascending node of the target orbit (Ω), the argument of periapse of the target orbit (ω), the ΔV of the impulsive LOI (ΔV_{LOI}), and the date of the LOI (t_{LOI}). Figures 4-4 and 4-5 show two examples of lunar transfers generated with this procedure using the inputs summarized in Table 4-2. Figure 4-4 illustrates a direct 4-day transfer and Fig. 4-5 illustrates an 84-day low-energy transfer.

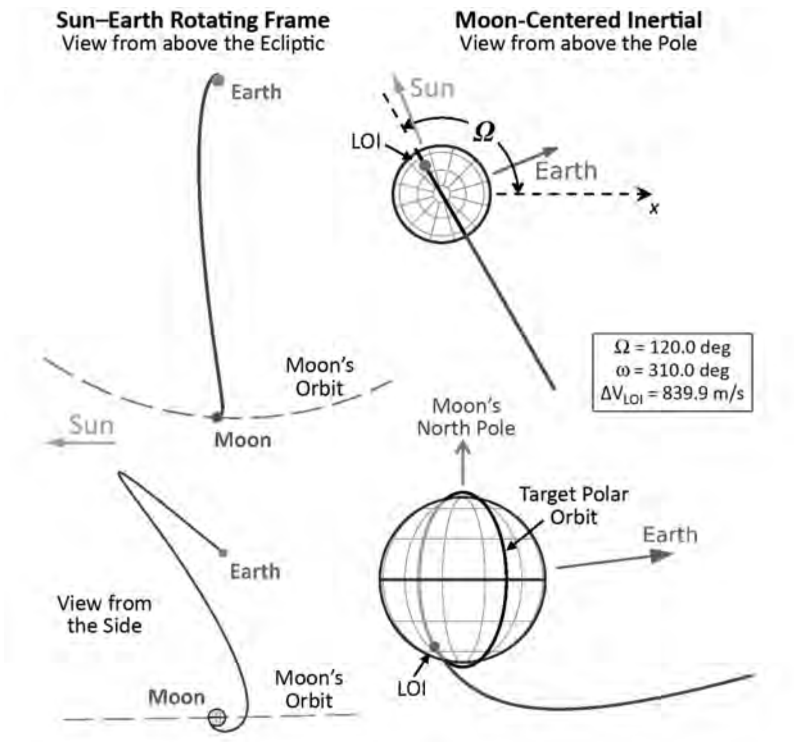

Figure 4-4 An example 4-day direct lunar transfer [2] (Copyright © 2011 by American Astronautical Society Publications Office, all rights reserved, reprinted with permission of the AAS).

All integrations performed here have been performed using a DIVA integrator (Section 2.7.1) with tolerance set to 1×10^{-10}; the force model includes the Sun, Earth, Moon, and each of the planets, all configured as point-mass gravitating bodies whose positions are estimated from JPL's DE421 Planetary and Lunar Ephemeris (Section 2.5.3).

Many surveys have been conducted, searching for practical lunar transfers. In general, a survey fixes the parameters Ω and t_{LOI} and systematically varies the other two parameters. This process generates a two-dimensional map displaying a parameter—typically the TLI altitude—which changes smoothly as either Ω or t_{LOI} shift. These surveys are described in more detail in the next sections.

4.4.2 Example Survey

Figure 4-6 shows the results of an example survey of lunar transfers. In this example, Ω is set to 120 deg, the LOI date is set to 18 July 2010 09:50:08 ET, the value

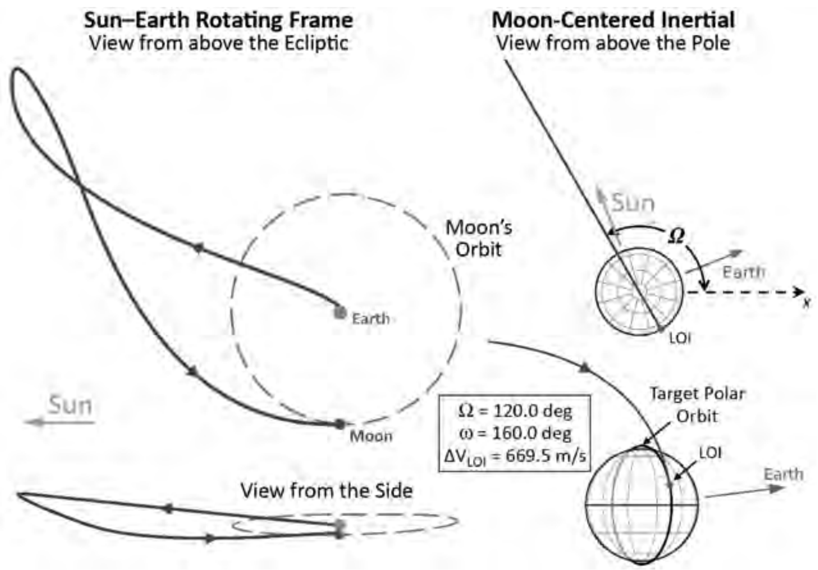

Figure 4-5 An example 84-day low-energy lunar transfer [2] (Copyright © 2011 by American Astronautical Society Publications Office, all rights reserved, reprinted with permission of the AAS).

Table 4-2 The inputs and performance parameters of the two example lunar transfers shown in Figs. 4-4 and 4-5. Both transfers begin in a 185-km circular low Earth orbit (LEO) parking orbit before their injections, and both transfers arrive at the Moon at a time t_{LOI} of 18 July 2010 9:50:08 ET.

Figure	Ω	ω	ΔV_{LOI}	Duration	LEO Inclination (deg)		C_3
#	(deg)	(deg)	(m/s)	(days)	Equatorial	Ecliptic	(km^2/s^2)
4-4	120.0	310.0	839.878	4.036	62.114	39.761	−2.064
4-5	120.0	160.0	669.543	83.706	28.093	5.921	−0.725

of ω is systematically varied from 0–360 deg, and ΔV_{LOI} is systematically varied from 650–1050 meters per second (m/s), a range empirically determined to generate practical transfers. Figure 4-6 shows the altitude of the trans-lunar injection point for each combination of ω and ΔV_{LOI}, assuming a spherical Earth with radius of 6378.136 km. The points shaded white correspond to trajectories that arrive at the Moon such that when propagated backward in time they never come any closer to the Earth than the orbit of the Moon itself. The points shaded black correspond to trajectories that arrive at the Moon such that when propagated backward in time they approach within 10,000 km of the Earth: trajectories that may be used to generate real missions [183, 184, 190, 191], assuming the departure time and geometry are

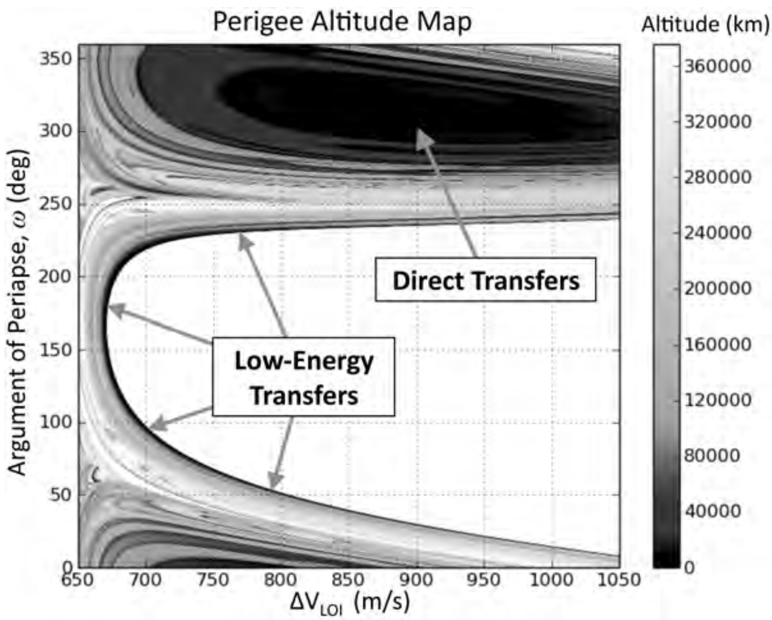

Figure 4-6 The altitude of the TLI location for each combination of ν and ΔV_{LOI}, given a lunar orbit insertion on July 18, 2010 into a lunar orbit with Ω equal to 120 deg [2] (Copyright © 2011 by American Astronautical Society Publications Office, all rights reserved, reprinted with permission of the AAS).

acceptable (see Section 6.5 for more information about generating a real mission using a ballistic guess).

The plot shown in Fig. 4-6 contains many interesting features. First, roughly half of the state space is white, corresponding to trajectories that arrive at the Moon from heliocentric orbits. With a quick investigation, one finds that the large black field toward the top of the plot corresponds to direct transfers to the Moon, that is, trajectories that take 2–12 days to reach the Moon, in family with the transfers that were implemented by the Apollo program and *LRO*; though most of the trajectories include Earth phasing orbits that extend the transfer's duration. The black curve that outlines the large white field corresponds to low-energy lunar transfers that require 80–120 days. There are many other curves throughout the plot that correspond to trajectories that enter some sort of large Earth orbit, or perform a combination of one or more flybys.

The direct transfers that are observed in the upper part of the plot shown in Fig. 4-6 require ΔV_{LOI} values from 760 m/s to 1000 m/s or more. The direct transfers that don't involve any Earth phasing orbits or any sort of lunar flyby require at least 818 m/s, though nearly all require 845 m/s or more. Figure 4-7 explores the structure of the direct transfer state space, presenting two additional maps that only show those

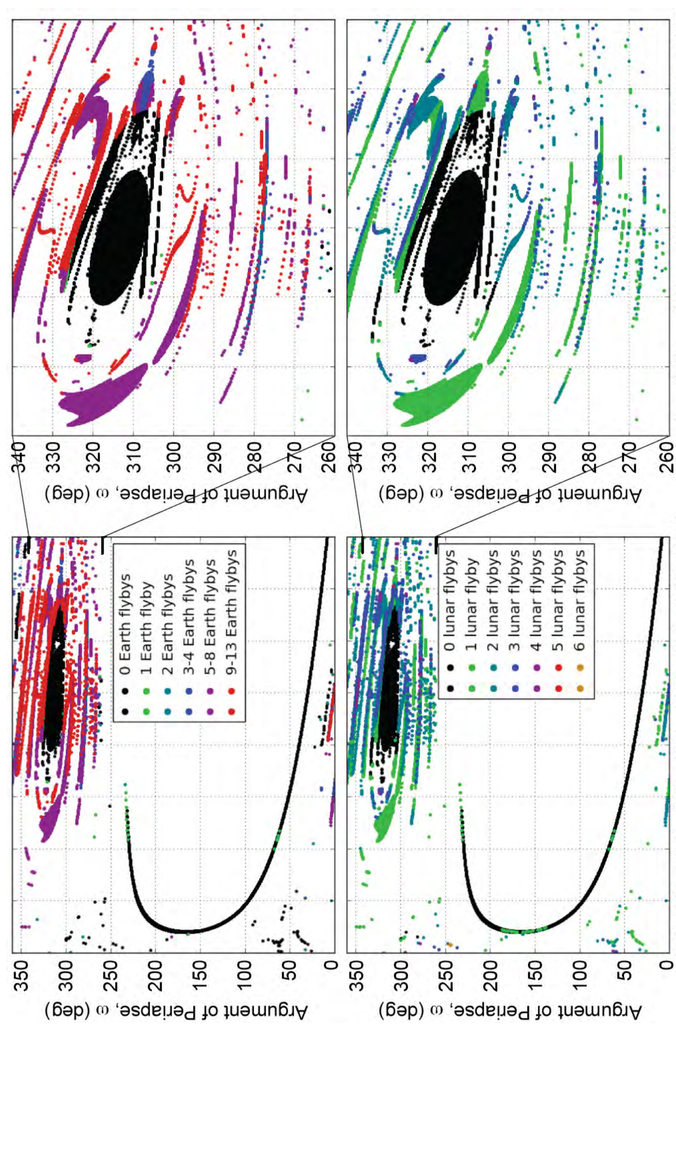

Figure 4-7 The transfers shown in Fig. 4-6 that approach within 1000 km of the Earth, shaded according to the number of Earth phasing orbits (top) and lunar flybys (bottom) that they make [2] (Copyright © 2011 by American Astronautical Society Publications Office, all rights reserved, reprinted with permission of the AAS). The upper portion of each map, corresponding with direct lunar transfers, is magnified in the plots on the right. *(See insert for color representation of this figure.)*

trajectories that approach within 1000 km of the Earth; the two maps are shaded according to the number of Earth perigee passages (top) and lunar flybys (bottom) that they make before arriving at their target orbit. One notices that direct transfers with more phasing orbits and/or lunar flybys may require less orbit insertion ΔV than the most basic lunar transfers. In any case, simple low-energy trajectories exist that require as little as 669 m/s, ∼100 m/s less than most multi-rev direct transfers observed and ∼170 m/s less than most simple direct transfers.

Tables 4-3 and 4-4 summarize the performance parameters of several example direct lunar transfers and low-energy lunar transfers, respectively. Several examples of these trajectories are shown in Figs. 4-8 and 4-9, respectively. One can see that the value of ΔV_{LOI} is generally over 100 m/s lower for low-energy transfers in nearly all examples, though the TLI injection energy, C_3, is higher. The injection energy of direct lunar transfers is very close to -2.0 km^2/s^2, compared to a value of approximately -0.7 km^2/s^2 for low-energy transfers. Both types of transfers include missions with a wide range of TLI inclinations, both relative to the Earth's Equator and to the ecliptic. This suggests that transfers can begin from any inclination about the Earth. Section 6.5 demonstrates that one can add one to three maneuvers and adjust a trajectory to depart from a specified TLI inclination rather than the ballistic inclination value shown in the tables for a very modest ΔV cost. The total ΔV required to make this adjustment is on the order of 1 m/s per degree of inclination change.

4.4.3 Arriving at a First-Quarter Moon

All of the transfers presented in the previous section arrive at the Moon at a particular time into a particular orbit, namely, a circular, polar orbit with a longitude of the ascending node, Ω, of 120 deg and a time of arrival, t_{LOI}, of 18 July 2010 at 9:50:08 ET. This time of arrival corresponds to a moment in time when the Sun–

Table 4-3 A summary of the performance parameters of several direct lunar transfers shown in Fig. 4-6 and illustrated in Fig. 4-8 [2] (Copyright © 2011 by American Astronautical Society Publications Office, all rights reserved, reprinted with permission of the AAS).

Traj #	Ω (deg)	ω (deg)	ΔV_{LOI} (m/s)	Duration (days)	LEO Inclination (deg) Equatorial	LEO Inclination (deg) Ecliptic	C_3 (km^2/s^2)	# Earth Flybys	# Moon Flybys
D1	120.0	321.3	818.0	4.111	22.147	8.551	−2.078	0	0
D2	120.0	326.4	860.4	4.155	43.459	62.667	−2.058	0	0
D3	120.0	304.8	867.5	4.004	85.516	63.963	−2.045	0	0
D4	120.0	301.5	947.7	3.942	142.173	123.280	−2.006	0	0
D5	120.0	311.7	971.8	4.009	131.320	154.340	−2.002	0	0
D6	120.0	321.0	813.3	13.941	24.717	6.435	−2.095	1	0
D7	120.0	326.4	868.0	14.005	52.683	72.504	−2.071	1	0
D8	120.0	279.0	870.0	32.759	19.407	31.944	−2.046	2	1
D9	120.0	325.5	758.0	67.175	37.135	13.784	−2.292	6	1
D10	120.0	327.0	810.1	84.747	62.694	39.723	−2.055	7	3
D11	120.0	354.9	828.8	85.441	75.489	54.465	−2.061	7	1
D12	120.0	268.2	861.4	141.341	46.894	63.333	−2.054	8	1

Table 4-4 A summary of the performance parameters of several low-energy lunar transfers shown in Fig. 4-6 and illustrated in Fig. 4-9 [2] (Copyright © 2011 by American Astronautical Society Publications Office, all rights reserved, reprinted with permission of the AAS).

Traj #	Ω (deg)	ω (deg)	ΔV_{LOI} (m/s)	Duration (days)	LEO Inclination (deg) Equatorial	Ecliptic	C_3 (km^2/s^2)	# Earth Flybys	# Moon Flybys
L1	120.0	169.2	669.3	83.483	29.441	6.129	−0.723	0	0
L2	120.0	103.8	692.1	85.287	25.688	34.778	−0.723	0	0
L3	120.0	70.2	743.9	93.598	57.654	74.955	−0.667	0	0
L4	120.0	225.3	716.0	93.621	134.322	112.840	−0.657	0	0
L5	120.0	99.9	697.5	110.060	83.127	61.624	−0.697	0	0
L6	120.0	186.9	673.2	122.715	23.941	3.088	−0.712	0	0
L7	120.0	61.5	660.4	143.360	18.624	35.412	−0.572	0	1
L8	120.0	59.7	651.3	129.422	73.143	96.544	−0.612	0	3
L9	120.0	36.3	661.5	144.417	146.592	138.491	−0.658	0	1
L10	120.0	348.6	675.1	155.107	36.598	16.583	−0.645	5	1
L11	120.0	262.2	656.1	141.982	153.641	176.867	−0.608	0	3
L12	120.0	244.2	657.8	136.687	179.084	156.890	−0.640	0	6

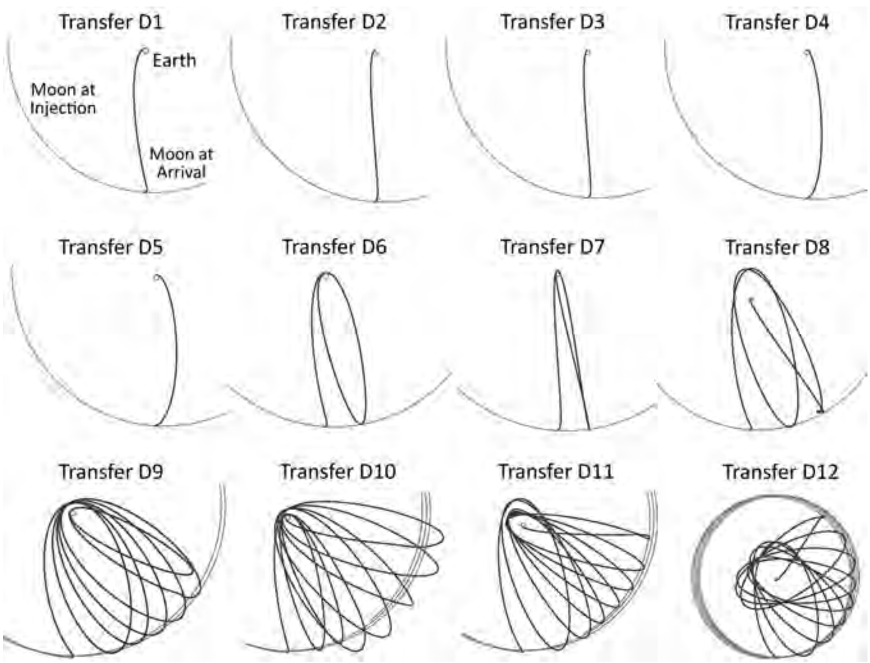

Figure 4-8 Example plots of several of the transfers summarized in Table 4-3. The trajectories are shown in the Sun–Earth rotating frame, such that the Sun is fixed on the x-axis toward the left [2] (Copyright © 2011 by American Astronautical Society Publications Office, all rights reserved, reprinted with permission of the AAS).

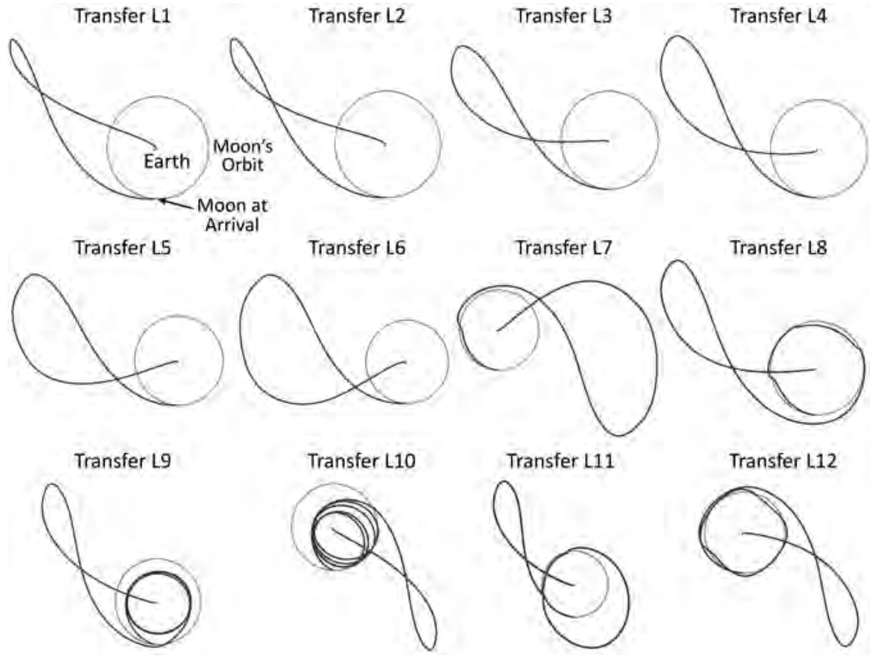

Figure 4-9 Example plots of several of the transfers summarized in Table 4-4. The trajectories are shown in the Sun–Earth rotating frame, such that the Sun is fixed on the x-axis toward the left [2] (Copyright © 2011 by American Astronautical Society Publications Office, all rights reserved, reprinted with permission of the AAS).

Earth–Moon angle is approximately equal to 90 deg at the Moon's first quarter. This is very similar to the arrival geometry of the two *GRAIL* spacecraft, though in a different month. In addition, the plane of the target orbit is nearly orthogonal to the Earth–Moon line. A polar orbit with an Ω-value of 111.9 deg (also 291.9 deg) is in a plane that is as close to orthogonal to the Earth–Moon line as a polar orbit can get on this date. The surveys presented in this section keep the time of arrival the same and explore the changes to the lunar transfers that occur as the target orbit's Ω-value is varied.

Figures 4-10 and 4-11 show surveys of the lunar transfer state space as Ω varies from 0–80 deg and 160–270 deg, respectively. There is a clear progression of the state space as Ω varies. Locations where direct and low-energy transfers exist are indicated. The state space varies much less discernibly when Ω is within $\sim$30 deg of 111.9 deg or 291.9 deg, namely, when the orbit is close to being orthogonal to the Earth–Moon line.

Many features are quickly apparent when studying the maps shown in Figs. 4-10 and 4-11. First, a large portion of each map is white, corresponding to combinations of ΔV_{LOI} and ω that result in trajectories that depart the Moon backward in time and

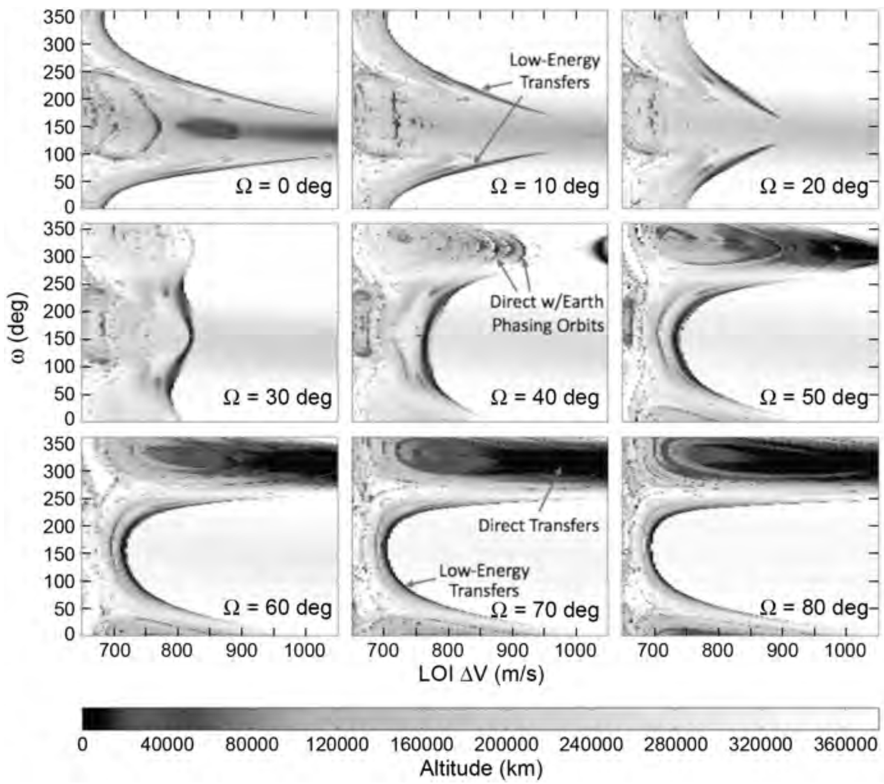

Figure 4-10 Nine surveys of trajectories that arrive at the first-quarter Moon, where the target orbit's Ω varies from 0–80 deg. Points in black originate from the Earth; other points are shaded according to how close they come to the Earth when propagated backward, using the light–dark shading scheme presented in Fig. 4-6 [2] (Copyright © 2011 by American Astronautical Society Publications Office, all rights reserved, reprinted with permission of the AAS).

traverse away from the Earth–Moon system. At lower ΔV_{LOI}-values, the trajectories depart the Moon backward in time and later impact the Moon or remain very near the Moon. One can see curves of black in each map, corresponding to trajectories that depart the Moon backward in time and eventually come very near the Earth; hence, making viable Earth–Moon transfers. The features are observed to shift in a continuous fashion across the range of Ω-values.

If one surveys these maps, one finds that low-energy transfers exist to any lunar orbit plane, but simple direct transfers only exist for certain ranges of Ω-values. Direct transfers can only reach orbits with Ω-values between approximately 50 deg and 170 deg and between approximately 230 deg and 350 deg for this particular

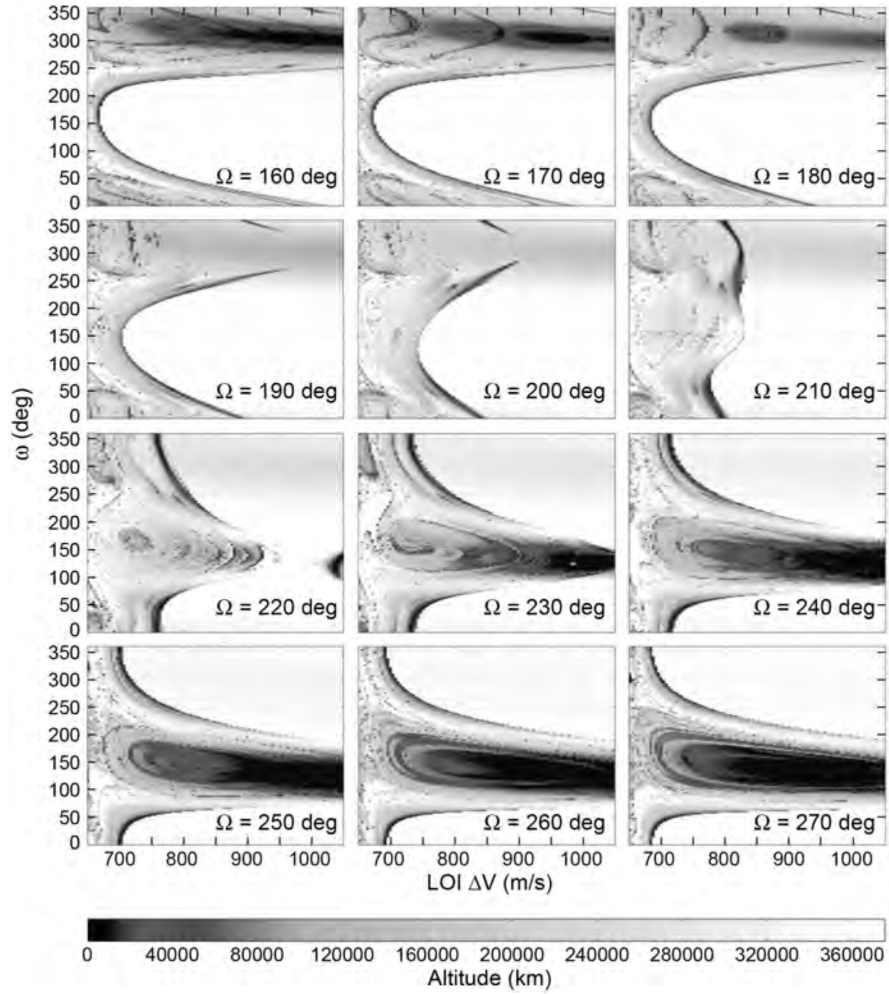

Figure 4-11 Twelve surveys of missions that arrive at the first-quarter Moon, where the target orbit's Ω varies from 160–270 deg. The maps are shaded according to the closest approach distance that the trajectories make with the Earth, as illustrated in Figs. 4-6 and 4-10 [2] (Copyright © 2011 by American Astronautical Society Publications Office, all rights reserved, reprinted with permission of the AAS).

arrival date. These orbit planes are within about 60 deg of being orthogonal to the Earth–Moon line; furthermore, direct lunar transfers require less ΔV for their orbit insertions the closer they are to being orthogonal to the Earth–Moon line.

Figure 4-12 captures the least-expensive ΔV_{LOI} for simple direct lunar transfers, as well as simple low-energy lunar transfers (that is, transfers that do not involve

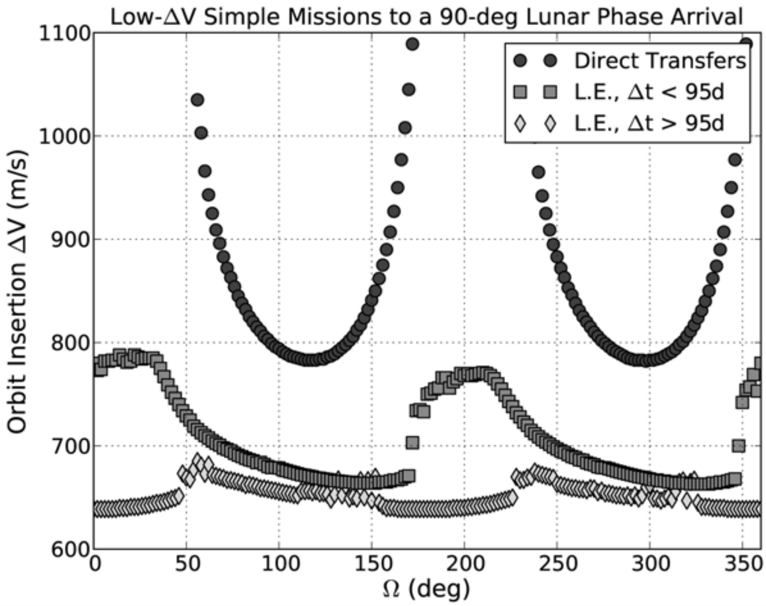

Figure 4-12 The minimum lunar orbit insertion ΔV for direct and low-energy (L.E.) lunar transfers, requiring no Earth phasing orbits nor lunar flybys for transfers to a first-quarter Moon. Polar orbits with Ω-values of 111.9 deg and 291.9 deg are very close to orthogonal to the Earth–Moon line on this arrival date [2] (Copyright © 2011 by American Astronautical Society Publications Office, all rights reserved, reprinted with permission of the AAS).

lunar flybys or Earth phasing orbits) for any target orbit plane studied. Three curves are presented: *direct* transfers involve transfers that require less than 40 days to achieve (most require less than 10 days), *fast low-energy* transfers require less than 95 days, and *long low-energy* transfers require more than 95 days to achieve. The transfer durations are not permitted to exceed 160 days in this study. There are many trajectories that require more ΔV than what is shown in Fig. 4-12; the illustration tracks the least expensive transfer in each case. Trajectories with Earth phasing orbits and/or lunar flybys may require even less ΔV, but those are not tracked here since there are so many paths that a spacecraft can take through the system. One observes that low-energy transfers do indeed reach any target orbit, though the insertion ΔV costs vary as the orbit plane changes. Direct lunar transfers are indeed limited to certain orbital planes, and they require at least 120 m/s more LOI ΔV than a low-energy transfer to the same orbit. Further, the cost of longer low-energy transfers remains very constant—within 50 m/s of ΔV—for any target lunar orbit plane.

The lunar transfers with the least LOI ΔV and no low Earth or lunar periapse passages have been identified for each combination of Ω and ω; their performance parameters are plotted in Fig. 4-13. The left plot shows a map of the LOI ΔV cost of these transfers; the plot on the right shows the corresponding transfer duration for

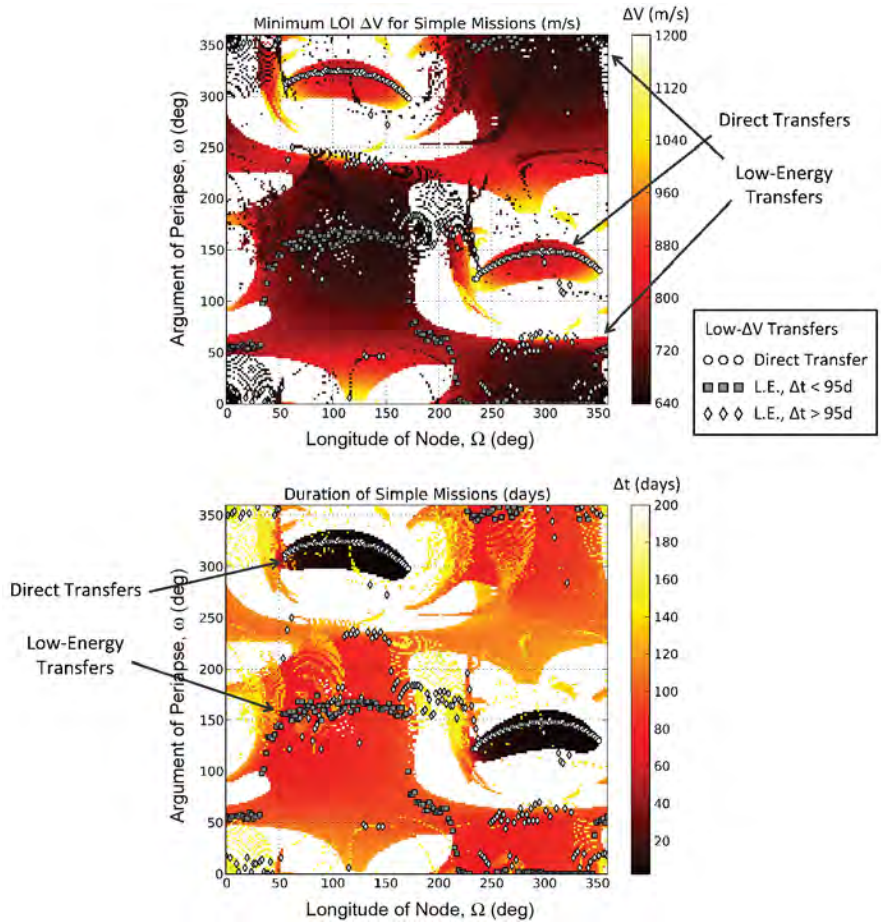

Figure 4-13 The combinations of Ω and ω that yield simple lunar transfers, that is, those without low Earth or lunar periapse passages. If multiple transfers exist for the same combination, then the one with the least LOI ΔV is shown. All of these transfers arrive at a first-quarter Moon. The low-ΔV transfers shown in Fig. 4-12 are indicated by dots in each map [2] (Copyright © 2011 by American Astronautical Society Publications Office, all rights reserved, reprinted with permission of the AAS). *(See insert for color representation of this figure.)*

each trajectory. The low-ΔV solutions identified in Fig. 4-12 are plotted in these maps for reference, and to identify their ω-values and durations. Direct transfers are easily discerned by observing the dark fields in the plot on the right, corresponding to short-duration transfers. One can see that there are large fields of combinations of Ω and ω that yield low-energy transfers, though the costs increase as one moves away from the low-ΔV curves. One can see that the combinations of Ω and ω that yield practical direct transfers are much more limited.

The maps shown in Fig. 4-13 are very useful: they illustrate what sorts of transfers may be used to reach any given polar orbit at the Moon, given that the transfers must arrive at the Moon at this particular arrival time. Missions that target an elliptical orbit must consider which argument of periapsis value to target; missions that aim to enter a circular orbit may likely use any ω for the initial orbit insertion, simplifying the trade space. Similar maps may be generated for any lunar arrival time: two different arrival times will be considered in the next sections.

4.4.4 Arriving at a Third-Quarter Moon

All of the transfers studied so far have arrived at the Moon at the same time, when the Moon is at its first quarter. Yet spacecraft missions may need to arrive at the Moon at any time of the month. As a second step in this survey, lunar transfers are studied that arrive at the Moon on 3 August 2010 at 04:38:29 ET: a time when the Moon has reached its third quarter. Figure 4-14 shows two example transfers that arrive at the third-quarter Moon, where the trajectory on the left is a direct lunar transfer and the trajectory on the right is a low-energy transfer. Neither transfer requires any extra Earth phasing orbits or lunar flybys. One notices that the low-energy transfer extends away from the Sun rather than toward it as seen in Figs. 4-5 and 4-9. Otherwise the transfers appear very similar to those studied previously. The symmetry observed here is expected according to the nearly symmetrical dynamics in the Sun–Earth system [86]. The Sun–Earth L_1 and L_2 points are located nearly the same distance from the Earth, and three-body libration orbits about those Lagrange points behave in a very similar fashion [46].

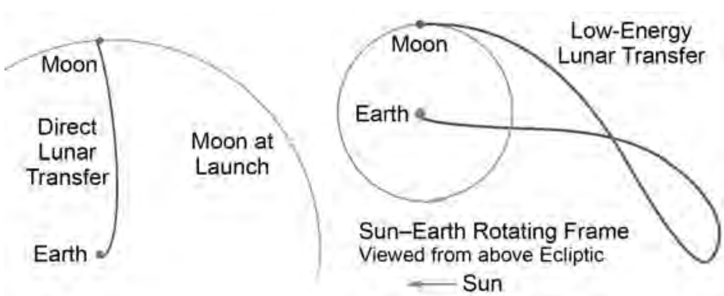

Figure 4-14 Two example lunar transfers that arrive at a third-quarter Moon. The transfers are simple, direct (left) and low-energy (right) lunar transfers with no Earth phasing orbits nor lunar flybys. The transfers are viewed from above in the Sun–Earth rotating frame of reference [2] (Copyright © 2011 by American Astronautical Society Publications Office, all rights reserved, reprinted with permission of the AAS).

One may construct state space maps for transfers to a third-quarter Moon in the same way that maps have been constructed previously to a first-quarter Moon. Figures 4-15 and 4-16 plot state space maps for transfers to target orbits with Ω-values of 0–80 deg and 180–260 deg, respectively. These ranges of Ω-values track the interesting features as the orbit plane changes; the maps of the Ω-values between those plotted in the figures vary little across the range. An orbit with an Ω-value of 126.9 deg (also 306.9 deg) is as close to orthogonal to the Earth–Moon axis as a polar orbit can be at this time. Transfers within about 60 deg of this angle are all very similar, though the cost of those transfers rises as the orbital plane moves away from this optimal Ω-value. When one compares the maps shown in Figs. 4-15 and 4-16 to those constructed earlier in Figs. 4-10 and 4-11, one sees that the maps are very

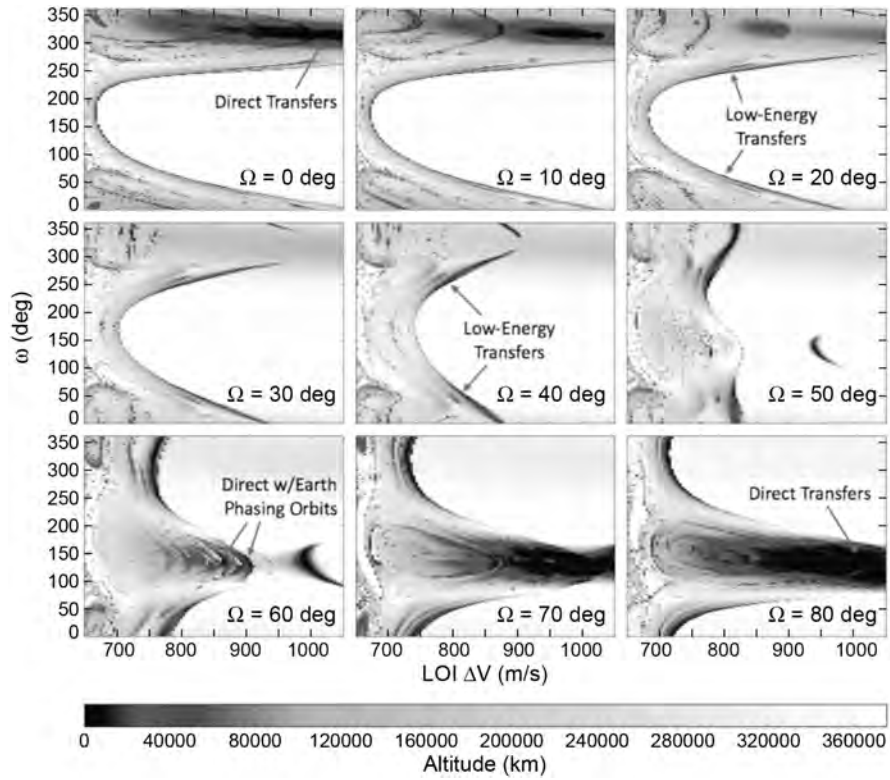

Figure 4-15 Nine surveys of missions that arrive at the third-quarter Moon, where the target orbit's Ω varies from 0–80 deg. The points are again shaded according to how close they approach to the Earth when propagated backward in time, using the same light–dark shading scheme applied in previous maps [2] (Copyright © 2011 by American Astronautical Society Publications Office, all rights reserved, reprinted with permission of the AAS).

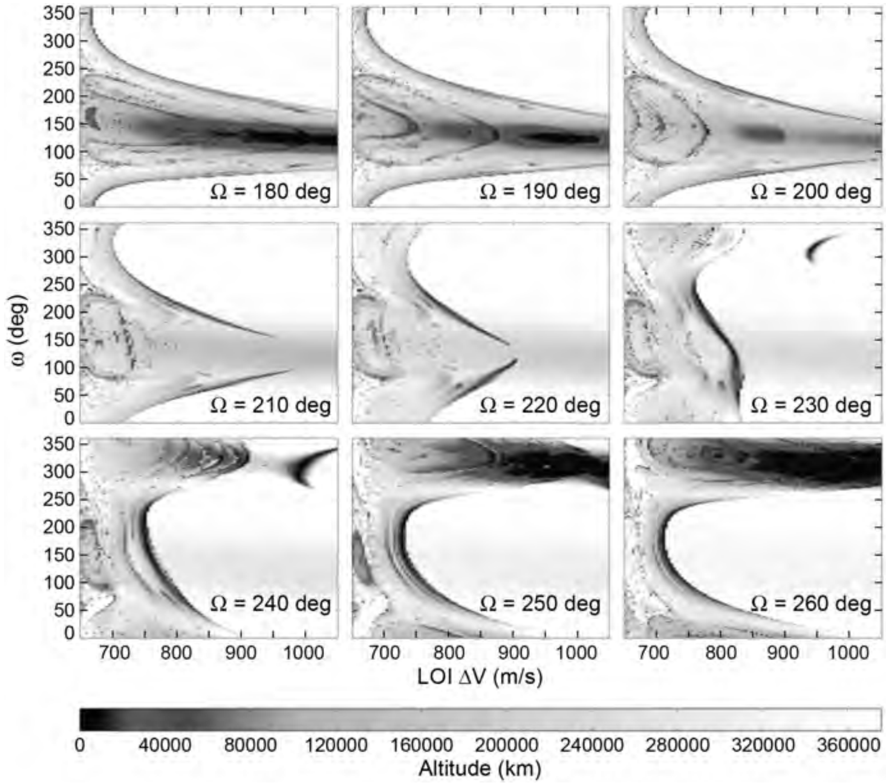

Figure 4-16 Nine surveys of missions that arrive at the third-quarter Moon, where the target orbit's Ω varies from 180–260 deg [2] (Copyright © 2011 by American Astronautical Society Publications Office, all rights reserved, reprinted with permission of the AAS).

similar with a 195 deg plane change. The transfers are arriving at the Moon when it is 180 deg further along in its orbit in the Sun–Earth synodic frame and 195 deg further in its orbit inertially, while the inertial coordinate axes that define Ω and ω have not changed.

Figure 4-17 shows the same two plots as shown in Fig. 4-13 for these third-quarter lunar arrival transfers. The maps show the LOI ΔV cost and transfer duration for simple lunar transfers that target different lunar orbits. As before, if there are multiple lunar transfers that may be used to arrive at the same lunar orbit, then the maps present the parameters for the transfer with the least LOI ΔV. The maps illustrate that the same trends exist to third-quarter lunar arrivals as do to first-quarter lunar arrivals, but with a 195-deg shift in Ω.

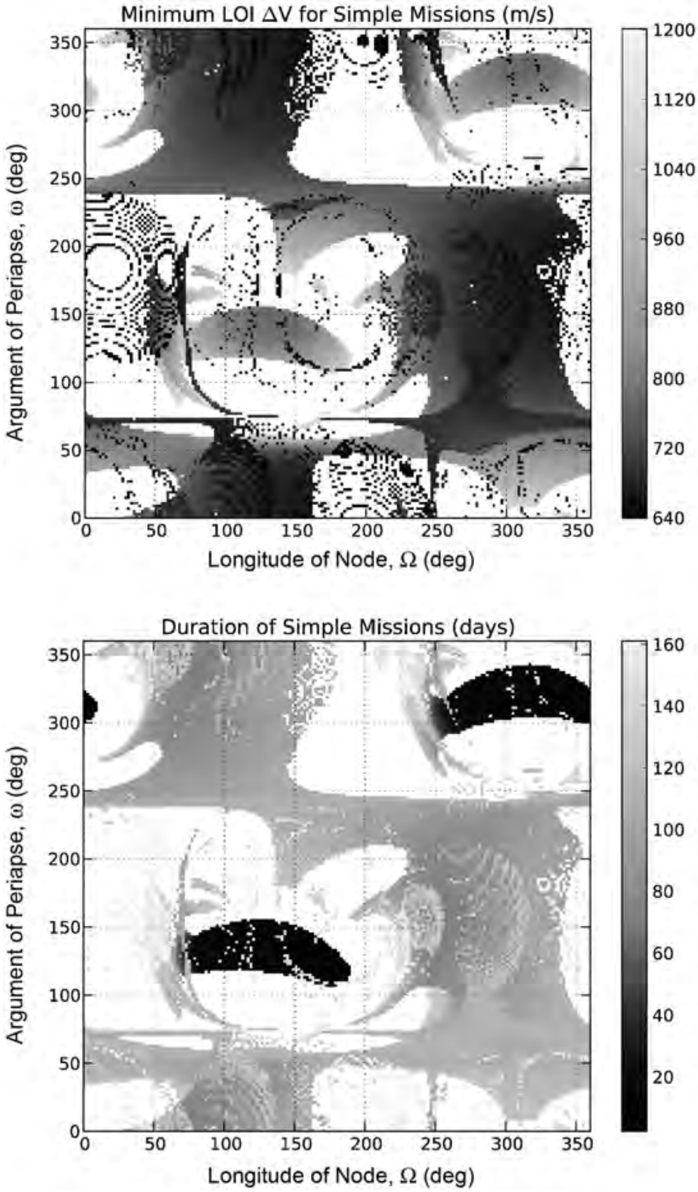

Figure 4-17 The combinations of Ω and ω that yield simple lunar transfers, that is, those without low Earth or lunar periapse passages. If multiple transfers exist for the same combination, then the one with the least LOI ΔV is shown. All of these transfers arrive at a third-quarter Moon [2] (Copyright © 2011 by American Astronautical Society Publications Office, all rights reserved, reprinted with permission of the AAS).

4.4.5 Arriving at a Full Moon

Trajectories have been studied that arrive at the Moon when the Sun–Earth–Moon angle is near 90 deg; this section briefly considers trajectories that arrive at a full Moon, when the Sun–Earth–Moon angle is approximately 180 deg. Lunar transfers that arrive at a new Moon have much the same characteristics as those that arrive at a full Moon, but with a familiar 180 deg $\pm$ 15 deg shift in Ω; for brevity they will not be shown here.

Figures 4-18 and 4-19 present state space maps for trajectories that arrive at the full Moon in polar orbits with Ω-values in the ranges 90–170 deg and 270–350 deg, respectively. The maps not shown vary only gradually between these maps. One observes that direct lunar transfers arrive at the full Moon with low-ΔV insertions at Ω-values approximately 90 deg apart from those that arrive at the first-quarter and

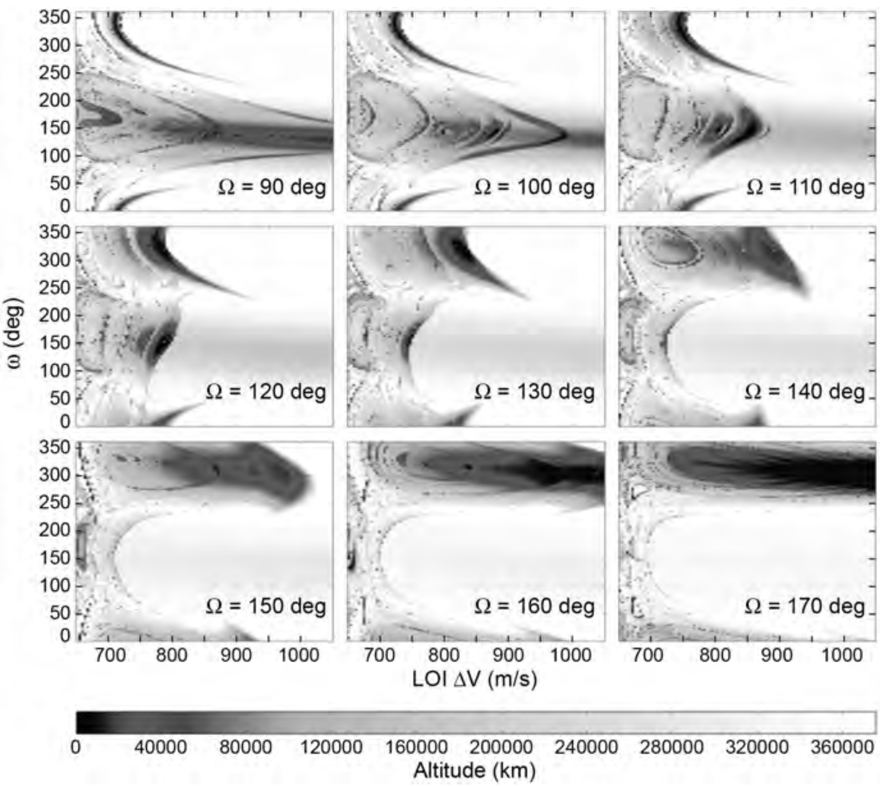

Figure 4-18 Nine surveys of missions that arrive at a full Moon, where the target orbit's Ω varies from 90–170 deg [2] (Copyright © 2011 by American Astronautical Society Publications Office, all rights reserved, reprinted with permission of the AAS).

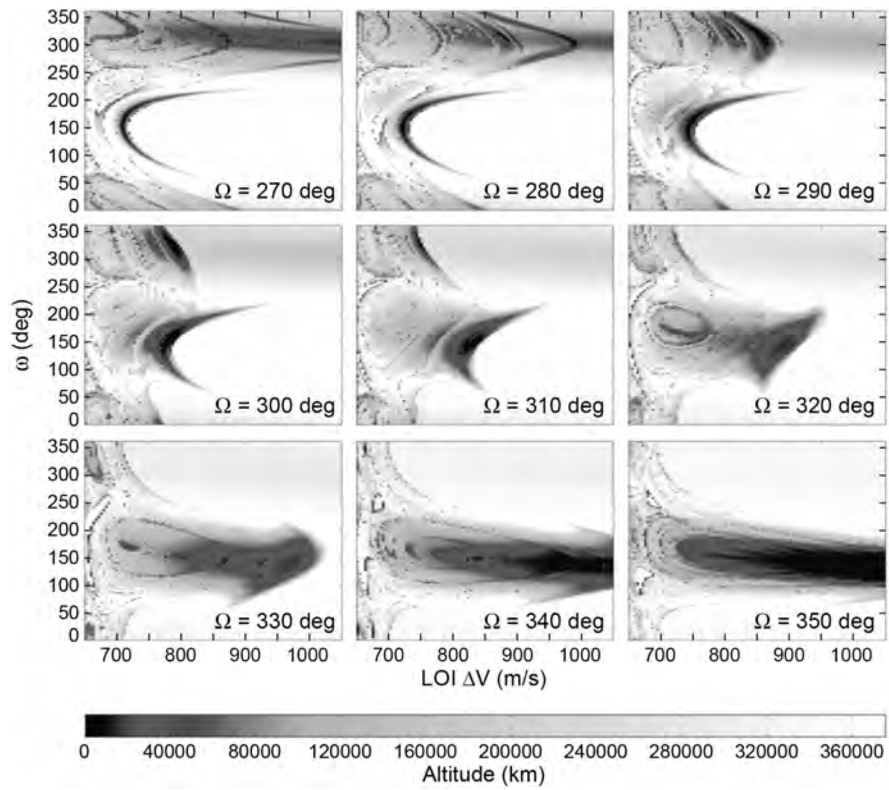

Figure 4-19 Nine surveys of missions that arrive at a full Moon, where the target orbit's Ω varies from 270–350 deg [2] (Copyright © 2011 by American Astronautical Society Publications Office, all rights reserved, reprinted with permission of the AAS).

third-quarter Moons. This demonstrates additional evidence that the minimum orbit insertion ΔV requirements for direct lunar transfers occurs when the orbit's plane is nearly orthogonal to the Earth–Moon line.

The low-energy lunar transfers' locations in the full-Moon state space maps evolve somewhat differently as Ω varies compared with their evolutions in the state space maps for first- and third-quarter Moons. Low-energy transfers still arrive at the Moon for any Ω-value, but the range of ω-values that may be used are bifurcated along the range of Ω-values. Many of the low-energy transfers that require the least LOI ΔV arrive at the full Moon at ω-values near 75 deg and 255 deg. These transfers fly further out of the plane of the Moon's orbit than others; those transfers that remain closer to the Moon's orbital plane require more ΔV and target ω-values near 165 deg and 345 deg. These characteristics are also apparent in Fig. 4-20, which shows the LOI ΔV and transfer duration state space maps for lunar transfers to this arrival time.

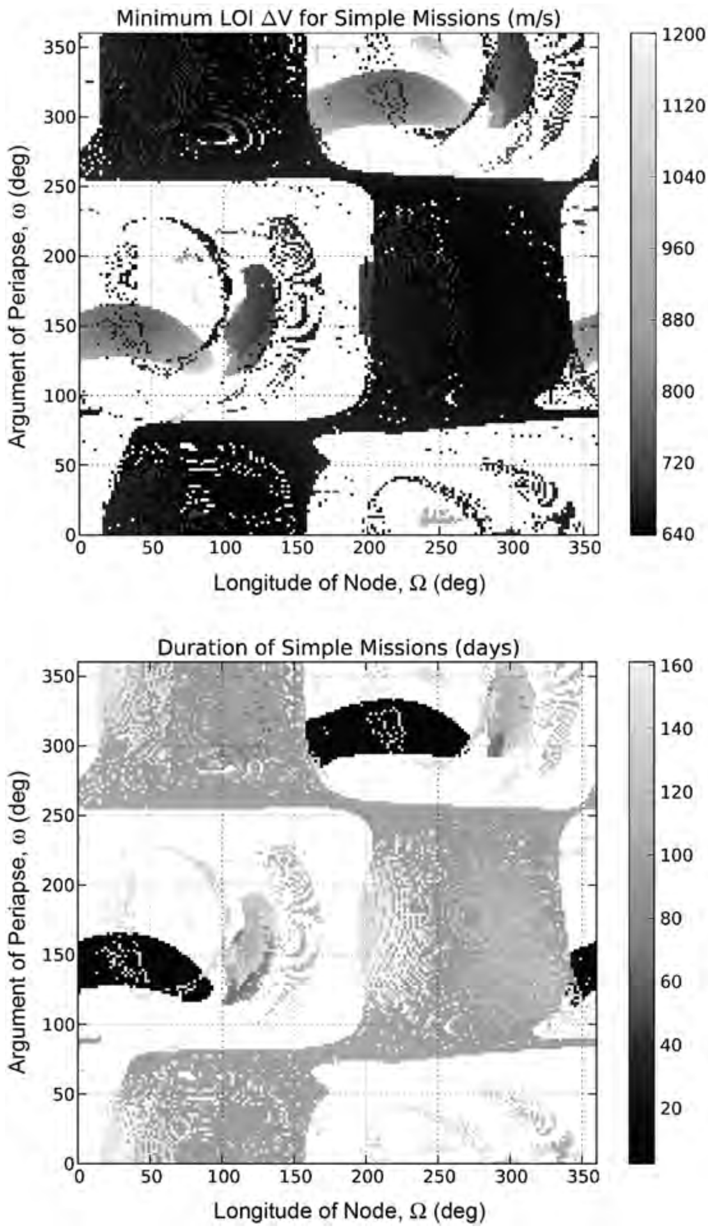

Figure 4-20 The combinations of Ω and ω that yield simple lunar transfers, that is, those without low Earth or lunar periapse passages. If multiple transfers exist for the same combination, then the one with the least LOI ΔV is shown. All of these transfers arrive at a full Moon [2] (Copyright © 2011 by American Astronautical Society Publications Office, all rights reserved, reprinted with permission of the AAS).

4.4.6 Monthly Trends

The work presented here describing transfers between the Earth and low lunar polar orbits has been extended, surveying transfers that arrive at the Moon at eight points in its orbit for several consecutive months. This section first presents results from surveys throughout one month and then considers similarities and variations that exist in lunar transfers across multiple months. The goal is to be able to predict the performance of lunar transfers for any given month.

Figure 4-21 shows eight state space maps, including those previously studied in Figs. 4-13, 4-17, and 4-20. These maps include simple transfers that arrive at the Moon at eight different points in a synodic month. Each map only tracks lunar transfers with no close lunar flybys or Earth phasing orbits, though each map does include both direct and low-energy transfers.

These maps are very useful to identify the combinations of Ω and ω that may be accessed via direct or low-energy transfers for a particular lunar arrival time. Similarly, the collection of these maps may be used to identify when to perform the lunar orbit insertion for a transfer to a particular combination of Ω and ω. One can see that low-energy transfers with LOI ΔV values below 700 m/s may be constructed that arrive at the Moon at any time. One also observes strong symmetry in the state space maps. First, each map shows a strong symmetrical mapping by shifting both Ω and ω by $\pm$180 deg. This shift corresponds to the difference between arriving at the Moon over the North Pole and arriving at the Moon over the South Pole. Second, a strong symmetry appears between two maps that correspond to arrivals $\pm$180 deg apart in the Moon's orbit: the maps show very similar characteristics when their arrival position and their Ω-values are both shifted by $\pm$180 deg. This shift corresponds to the symmetry that exists in the Sun–Earth three-body system: the dynamics are very similar, with a 180 deg rotation about the Earth, for the case where a spacecraft traverses from the Earth toward the Sun and for the case where a spacecraft traverses away from the Sun.

Figure 4-22 shows eight scatter plots, corresponding to the same arrival times presented in Fig. 4-21. The plots illustrate the relationships between each transfer's duration and its lunar orbit insertion ΔV. One can clearly see that direct transfers and low-energy transfers exist at every arrival time: direct transfers are shown on the far left of each plot, corresponding with short transfer durations and raised LOI ΔV requirements; low-energy transfers are similarly shown toward the bottom-right of each plot, corresponding with longer transfers and lower LOI ΔV requirements. Intermediate transfers exist for some arrival times, with transfer durations on the order of 60 days. One can see the same symmetry described above, between a given plot and the one that corresponds to a lunar arrival $\pm$180 deg apart. These plots are useful to quickly identify the limits of transfer duration and LOI ΔV for each type of transfer at any given lunar arrival time.

Most characteristics of ballistic two-burn lunar transfers repeat from one month to the next. The Moon's orbital plane is nearly coplanar to the Earth's, and the orbits of the bodies involved are nearly circular. However, since these conditions are not perfectly met, the characteristics of these lunar transfers do vary from one month

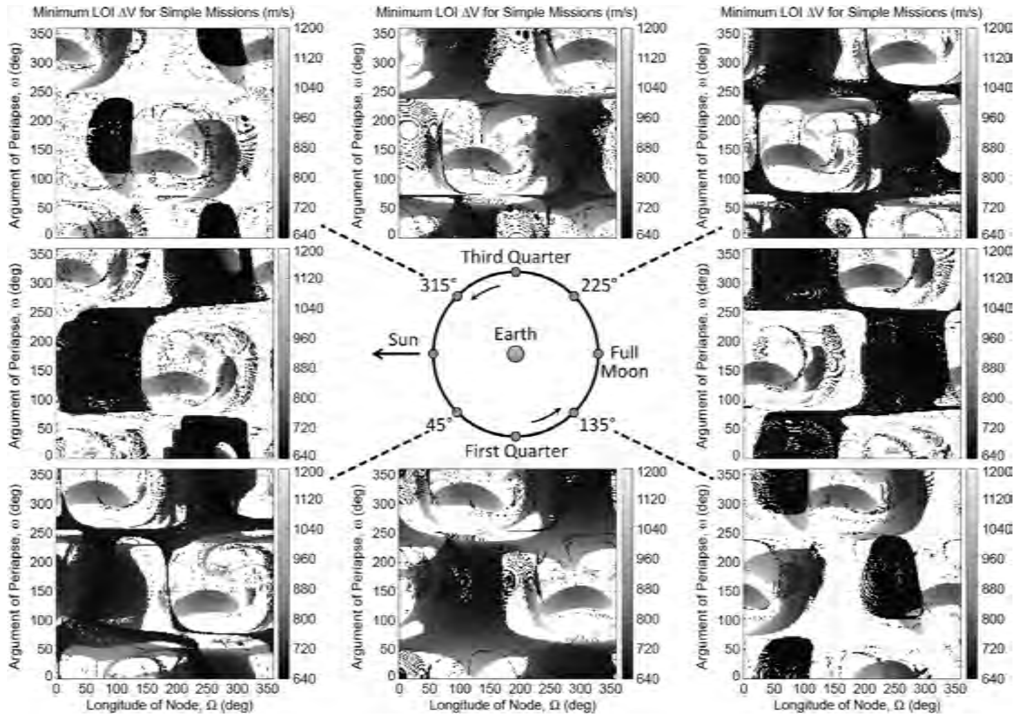

Figure 4-21 State space maps, illustrating the LOI ΔV required to transfer from the Earth to low polar lunar orbit at different arrival times in a month [2] (Copyright © 2011 by American Astronautical Society Publications Office, all rights reserved, reprinted with permission of the AAS).

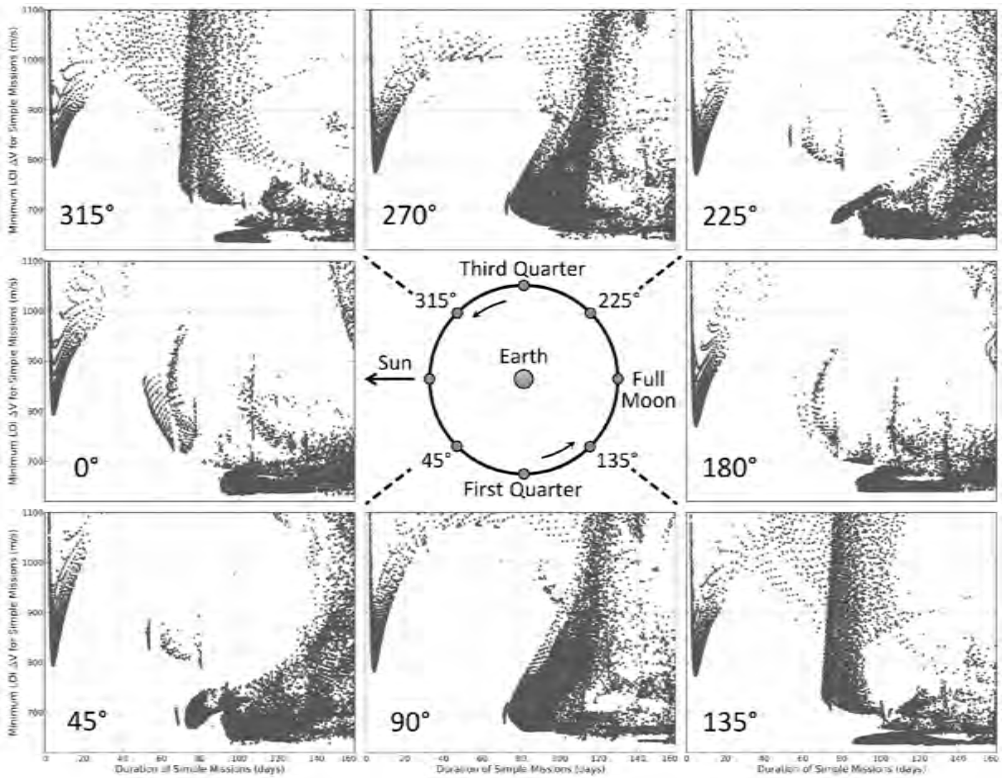

Figure 4-22 Scatter plots showing the relationship between transfer duration and a transfer's LOI ΔV cost for simple lunar transfers arriving at the Moon at different points about its orbit.

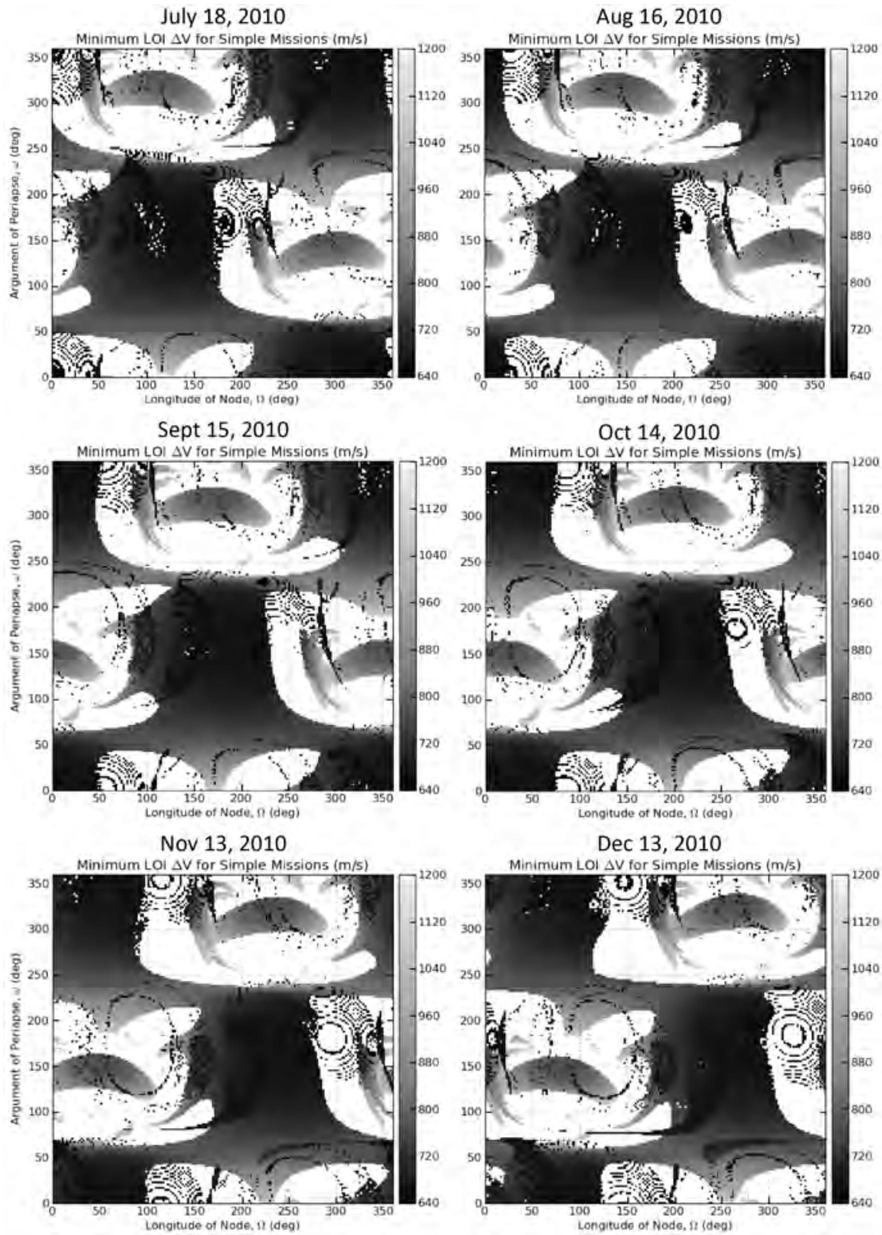

Figure 4-23 State space maps that illustrate the LOI ΔV for transfers to each combination of Ω and ω that arrive at the Moon at its first quarter in each of six consecutive months [2] (Copyright © 2011 by American Astronautical Society Publications Office, all rights reserved, reprinted with permission of the AAS).

to the next. The Moon's orbital plane and equatorial plane are tilted approximately 5.1 deg and 1.5 deg, respectively, relative to the ecliptic. One may therefore assume that the characteristics observed in the state space maps presented here vary by several degrees in their ω-values in a given month. The inclination of the trans-lunar departure state for a given type of lunar transfer may vary by many degrees from one month to the next, particularly on account of the obliquity of the Earth's spin axis.

It has been found that most types of *simple* lunar transfers appear in any given month and their characteristics remain relatively constant relative to the ecliptic. Figure 4-23 illustrates how little the state space maps vary from one month to the next, when evaluating simple lunar transfers. The six state space maps shown capture the LOI ΔV for transfers to each combination of Ω and ω that arrive at the Moon at its first quarter in each of six consecutive months. The only major apparent variation is that the features in each map shift approximately 30 deg in Ω from one month to the next. This is because Ω is defined inertially and the Earth moves approximately 30 deg in its orbit from one month to the next, rotating the Sun–Earth geometry. The more complex lunar transfers, such as those with multiple lunar flybys, vary much more on a monthly basis and may not even appear at all in a given month.

4.4.7 Practical Considerations

The surveys presented here study trajectories that are entirely ballistic—they do not contain any correction maneuvers or targeting maneuvers of any sort. When propagated backward in time from the Moon, if a trajectory arrives at the Earth without impacting the Moon, then it is considered a viable Earth–Moon transfer. However, the trajectory may have arrived at the Earth with an inclination that is unsuitable for a mission that launches from a particular launch site. Ideally, a mission would start in a low-Earth parking orbit with an inclination very close to that of the latitude of the launch site, for example, near 28.5 deg for missions that launch from Cape Canaveral, Florida. It is undesirable to perform a large plane change during launch and trans-lunar injection. Section 6.5 shows that one can add 1–3 small trajectory correction maneuvers to depart the Earth from a particular LEO parking orbit and transfer onto a desirable *low-energy* transfer to the Moon; and doing so requires only about 1 m/s per degree of inclination change. This works for low-energy transfers particularly well since low-energy transfers travel far from the Earth and spend many weeks doing so. This method does not work well for direct lunar transfers, which require far more ΔV to change planes.

Mid-course maneuvers may also be implemented to establish a launch period for a low-energy transfer to the Moon, extending or shrinking its transfer duration. Missions that implement direct lunar transfers may establish a launch period using Earth phasing orbits, making those sorts of transfers more desirable in the surveys presented here.

4.4.8 Conclusions for Low-Energy Transfers Between Earth and Low Lunar Orbit

The surveys presented in this section characterize two-burn lunar transfers that arrive at the Moon, targeting 100-km polar orbits with any orientation. Transfers are studied that arrive at an example first-quarter Moon, an example full Moon, and an example third-quarter Moon. Additional results are also presented for transfers that arrive at eight different times during a month and for several consecutive months. Many types of transfers are observed, including low-energy transfers, short-duration direct transfers, and variations that involve any number of lunar flybys and Earth phasing orbits, provided that they do not involve any deterministic maneuvers. The only two burns considered are the trans-lunar injection maneuver and orbit insertion maneuver.

It has been found that lunar transfers consistently require trans-lunar injection C_3 values on the order of -2.0 km^2/s^2 for direct transfers and -0.6 km^2/s^2 for low-energy transfers. Simple transfers typically require 2–12 days for direct transfers and 70–120 days for low-energy transfers, though both types can require more time. The low-energy transfers that require the least LOI ΔV require 640 m/s, or more depending on the target orbit and the arrival time; direct lunar transfers require at least 120 m/s more ΔV than low-energy transfers to the same arrival conditions. Further, low-energy transfers can reach many arrival conditions that direct transfers cannot reach without additional maneuvers. Practical simple direct transfers only exist that target a lunar orbit that is within 60 deg of being orthogonal to the Earth–Moon line, though the ΔV cost rises significantly when the orbit is beyond 30 deg of orthogonal. Low-energy transfers can target polar orbits with any argument of periapse, ω, or with any longitude of ascending node, Ω; targeting one such parameter restricts the other for a particular arrival date as illustrated in the state space maps presented here.

4.5 TRANSFERS BETWEEN LUNAR LIBRATION ORBITS AND LOW LUNAR ORBITS

Many mission designs may benefit by transferring a spacecraft from the Earth to a lunar libration orbit prior to descending to a low lunar orbit. For instance, Hill et al. [11], designed a mission where two satellites transferred to a halo orbit about the lunar L_2 point. One satellite remained there as a navigation and communication relay and the other satellite transferred to a low lunar orbit. Information about such transfers is summarized in Section 3.5.1 on page 224.

4.6 TRANSFERS BETWEEN LOW LUNAR ORBITS AND THE LUNAR SURFACE

Many historical missions have performed maneuvers to transfer a spacecraft from a low lunar orbit to the lunar surface, for example, the Apollo missions [1]. A few spacecraft, including Apollo missions, have then risen from the lunar surface and returned to lunar orbit. These maneuvers are very straightforward and may even be well approximated by conic sections; nevertheless, it is useful to briefly describe their designs here.

Let's assume we have a spacecraft in a 100-km circular lunar orbit. That spacecraft is traveling approximately 1633.5 m/s in its orbit and revolves about the Moon once every 117.8 minutes. The minimum ΔV required to place the spacecraft on a collision course with the Moon would reduce the spacecraft's orbital periapse to an altitude of 0 km, at which point it would just graze the surface, that is, a Hohmann transfer. This transfer requires a ΔV of approximately 23 m/s, sending the spacecraft on a 180-deg transfer in about 56.5 minutes. The spacecraft's grazing velocity upon arriving at its orbital periapse is approximately 1703.2 m/s. If the spacecraft performs a larger braking burn from its 100 km orbit, then its transfer orbit will strike the surface of the Moon at a steeper flight path angle in less time.

Figure 4-24 illustrates the flight path angles that may be achieved at the mean radius of the Moon as a function of the de-orbit burn ΔV for trajectories starting from an altitude of 100 km. One can see that a ΔV of 23 m/s is indeed required to obtain a flight path angle of 0 deg, which is the limit of trajectories that have a passive abort option, not including local geometry variations. Of course, by performing a braking burn ΔV of 1633.5 m/s, the spacecraft completely removes its orbital velocity and falls straight down to the surface, achieving a vertical impact.

Figure 4-25 illustrates the velocities that the spacecraft will have at the impact point, assuming the impact point occurs at a radius of 1737.4 km, for example, the mean radius of the Moon. Figure 4-26 shows the duration of time required to reach the impact point.

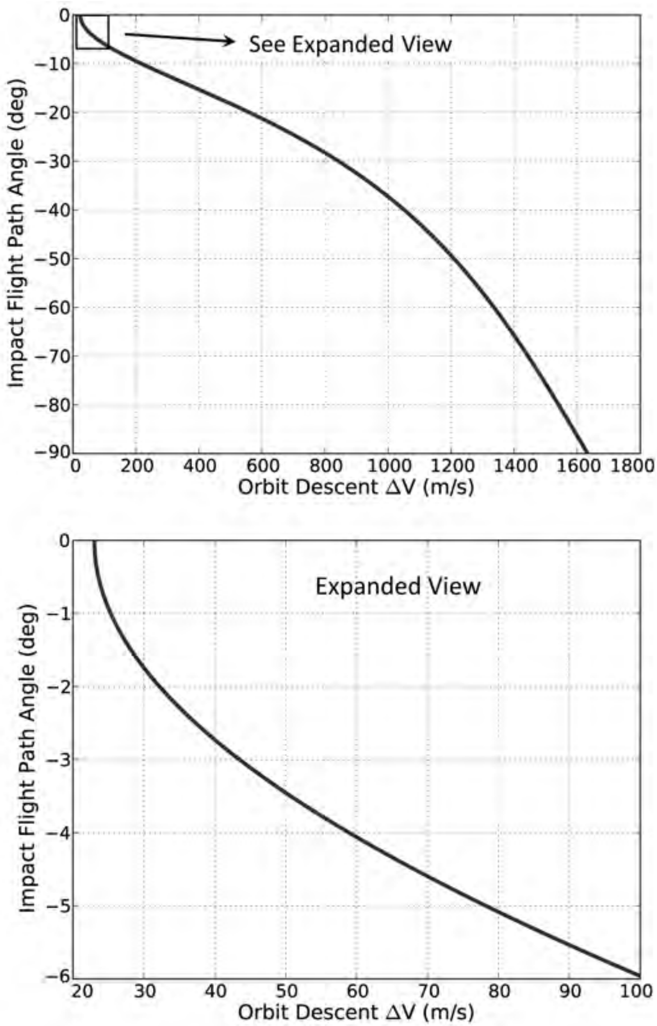

Figure 4-24 The flight path angles that may be achieved at the mean surface of the Moon as a function of the de-orbit burn ΔV for trajectories starting from a circular orbit at an altitude of 100 km.

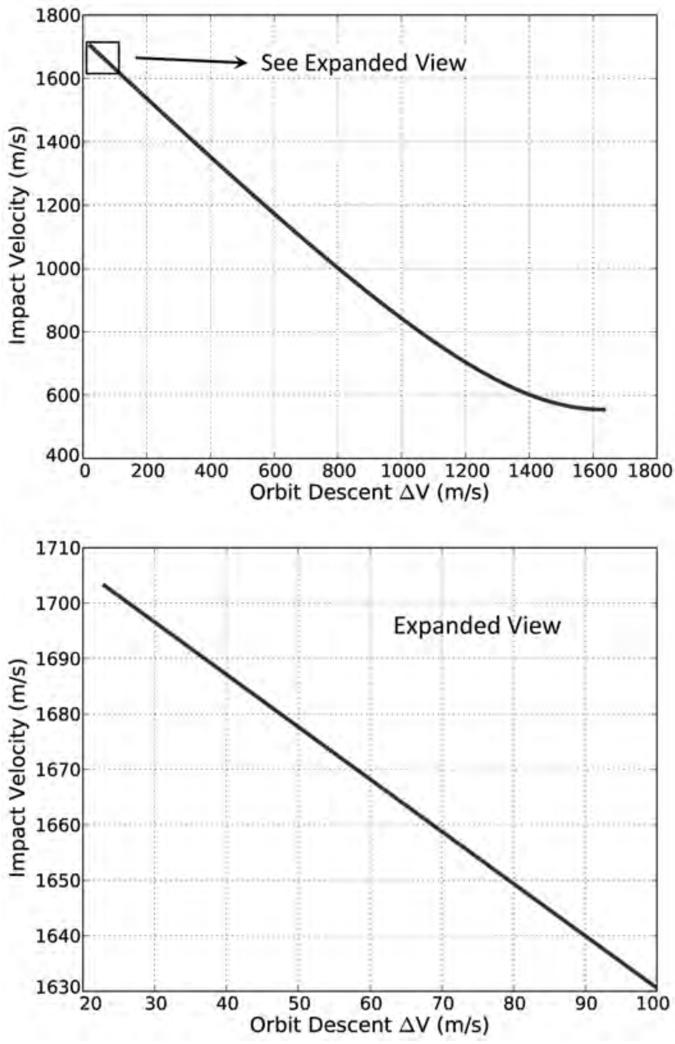

Figure 4-25 The impact velocity values that may be achieved at the mean surface of the Moon as a function of the de-orbit burn ΔV for trajectories starting from a circular orbit at an altitude of 100 km.

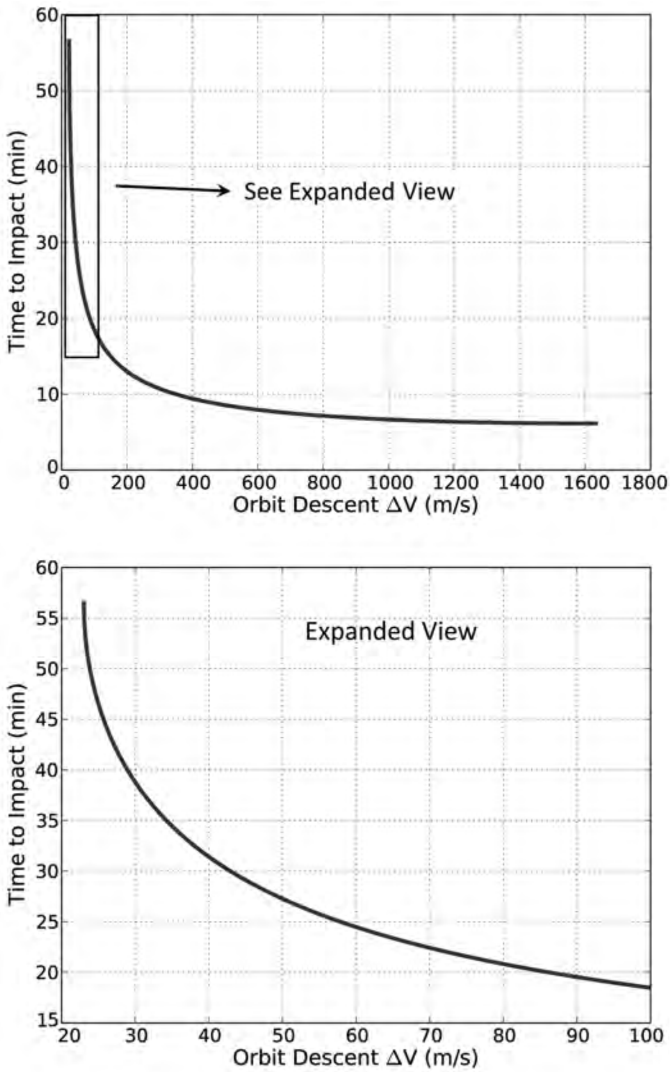

Figure 4-26 The duration of time required to reach the mean surface of the Moon as a function of the de-orbit burn ΔV for trajectories starting from a circular orbit at an altitude of 100 km.