XXV. Communications Systems Research: Communication and Tracking

A. Optimum Modulation Indices for Single-Channel, One-Way and Two-Way Coherent Communication Links

W. C. Lindsey

1. Introduction

In the design of two-way communication links (*SPS 37-31, -32, -33, -34, -35*, Vol. IV, pp. 374–379, 284–288, 290–296, 242–247, and 339–341, respectively), it is necessary to know how the total transmitter power should be allocated between the transmitted carrier and one or more subcarriers. Results presented herein permit the ratio of the power in the carrier to the total power transmitted, say m^2, to be specified so that the system error probability is a minimum. Results are given for both one-way and two-way links.

2. Model

In the following discussion, the notation and results reported in the above issues of the *SPS* shall be utilized. Briefly, a two-way coherent communication link may be defined as one which radiates a phase-modulated RF carrier from the Earth to the spacecraft; the spacecraft coherently tracks the noise-corrupted carrier component by means of a phase-locked loop. The phase-locked loop estimate of the frequency and phase of the observed carrier-component is used (after appropriate frequency translation) as a carrier for the transmission of telemetry back to Earth.

Since carrier tracking is noisy, the phase-locked loop estimate of the phase possesses a random perturbation which affects the performance of the ground-based telemetry demodulator. Thus, the observed data on Earth is perturbed by both the uplink and downlink additive noise. Consequently, the modulation index, which minimizes the error probability for the downlink, is generally different from that of the uplink.

It was shown (*SPS 37-33*) that $P_E(2)$, the bit error probability in an uncoded biphase-modulated telemetry system for the ground system telemetry demodulator, is given by

$$P_E(2) = \frac{1}{\pi}\int_0^{\pi} \frac{I_0(|\alpha_1 + \alpha_2 \exp(j\phi)|)}{I_0(\alpha_1)\, I_0(\alpha_2)}\, d\phi \int_{(2R_D)^{1/2}\cos\phi}^{\infty} \frac{1}{(2\pi)^{1/2}} \exp(-y^2/2)\, dy \qquad (1)$$

where R_D is the signal-to-noise ratio (SNR) in the data; α_1, the SNR in the spacecraft-carrier tracking loop; α_2 is the SNR in the ground receiver carrier-tracking loop; ϕ is the ground system phase error; and $I_0(x)$ is the imaginary Bessel function of zero order and of argument x. The bit error probability for the command link (uplink) is given by

$$P_E(1) = \lim_{\alpha_1 \to \infty} P_E(2) \tag{2}$$

More precisely, the basic parameters given in Eq. (1) are:

$R_{D2} = (1 - m_2^2) P_2 T_{b2} (1 - \lambda_2)/N_{02} = (1 - m_2^2) R_2$

$m_2^2 = P_{c2}/P_2 = \dfrac{\text{power in carrier component}}{\text{total power transmitted}}$

T_{b2} = duration of downlink's RF phase modulation

λ_2 = signal cross-correlation coefficient
= -1 for biphase modulation

N_{02} = noise spectral density perturbing the downlink, assumed white and single-sided

$\alpha_2 = 2m_2^2 P_2/N_{02} B_{L2} = m_2^2 \delta_2 R_2$

$\alpha_1 = \dfrac{2m_1^2 P_1}{N_{01} B_{L1}} K^{-1}(\beta)$

P_1 = total power transmitted on up-link

N_{01} = noise spectral density perturbing the uplink, assumed white and single-sided

B_{L1} = bandwidth of the carrier-tracking loop in the spacecraft

B_{L2} = bandwidth of the carrier-tracking loop in the ground receiver

$$\delta_2 = \frac{2\mathcal{R}_2}{B_{L2}(1-\lambda_2)} \quad \frac{1}{T_{b2} B_{L2} (1-\lambda_2)}$$

$$K(\beta) = \frac{2G^2}{3}\beta\left[\frac{(1+\beta)^2 - \beta + \beta^2/2}{(1+\beta)^3 - 2\beta(1+\beta)}\right]; \qquad \beta = \frac{B_{L1}}{B_{L2}}$$

$\mathcal{R}_2 = \dfrac{1}{T_{b2}}$ = downlink data rate

and G is the static phase gain and is determined by the ratio of the output frequency to the input frequency. In practice $K(\beta)$ is approximately one.

The integrals in Eqs. (1) and (2) may be evaluated; however, the desire to place the result into closed form seems, at this point, to be unrewarding. In any case, it is possible to show that Eqs. (1) and (2) may be written as

$$P_E(2) = \frac{1}{2}\left\{1 - \left(\frac{2R_n[1-m_n^2]}{\pi}\right)^{1/2} \exp\left[-\frac{R_n}{4}(1-m_n^2)\right] \times \sum_{k=0}^{\infty} (-1)^k \varepsilon_k b_{2k+1}(n) \frac{I_k\left[\frac{R_n}{4}(1-m_n^2)\right]}{1-4k^2}\right\}$$

$$n = 1, 2 \tag{3}$$

where

$$b_{2k+1}(n) = \prod_{i=1}^{n} \frac{I_k(\alpha_i)}{I_0(\alpha_i)}$$

and $\varepsilon_k = 1$ if $k = 0$, and $\varepsilon_k = 2$, if $k \neq 0$.

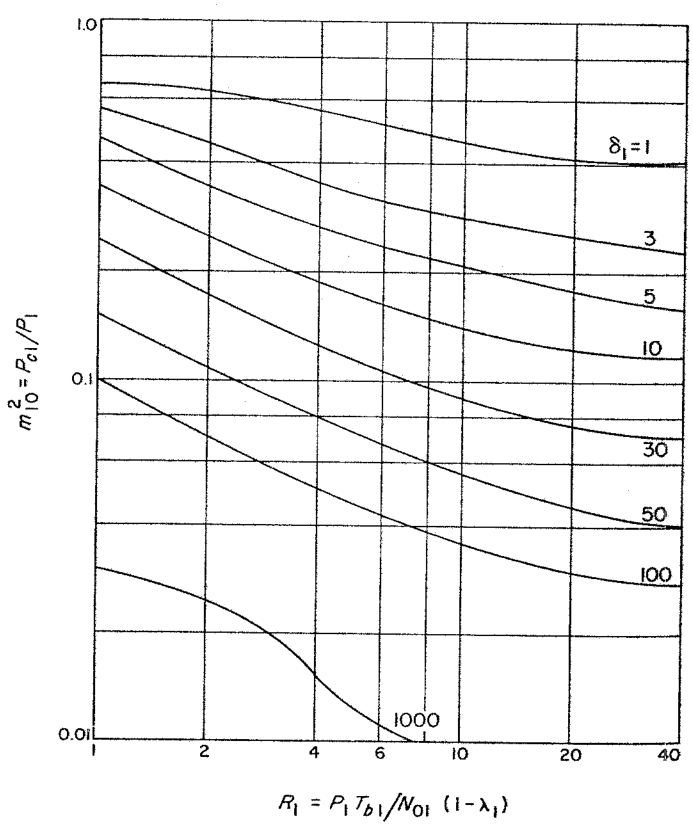

Fig. 1. Optimum modulation index vs R_1 for various values of δ_1

3. Results

It is quite apparent that any attempt to find the value of m_n, which minimizes $P_E(n)$ by the method of differentiation, presents formidable difficulties; however, the surface generated by Eq. (3) has been studied recently on the IBM 7090 Computer.

To illustrate the results graphically, consider the one-way link, i.e., $\alpha_1 = \infty$. Fig. 1 represents a plot of the parameters R_1 and the value of m_1 which minimize, say $m_{10}^2 = P_{c1}/P_1$, $P_E(1)$ for various values of the parameter $\delta_1 = 2\mathcal{R}_1/B_{L1}(1 - \lambda_1)$. These results have been obtained by fixing R_1 and searching, on the IBM 7090 computer, for that value of m_1 which minimizes $P_E(1)$, say $P_{E_0}(1)$. The corresponding plot of $P_{E_0}(1)$, i.e., minimum error probability, versus R_1 is given in Fig. 2. These two figures may be used in carrying out a particular design.

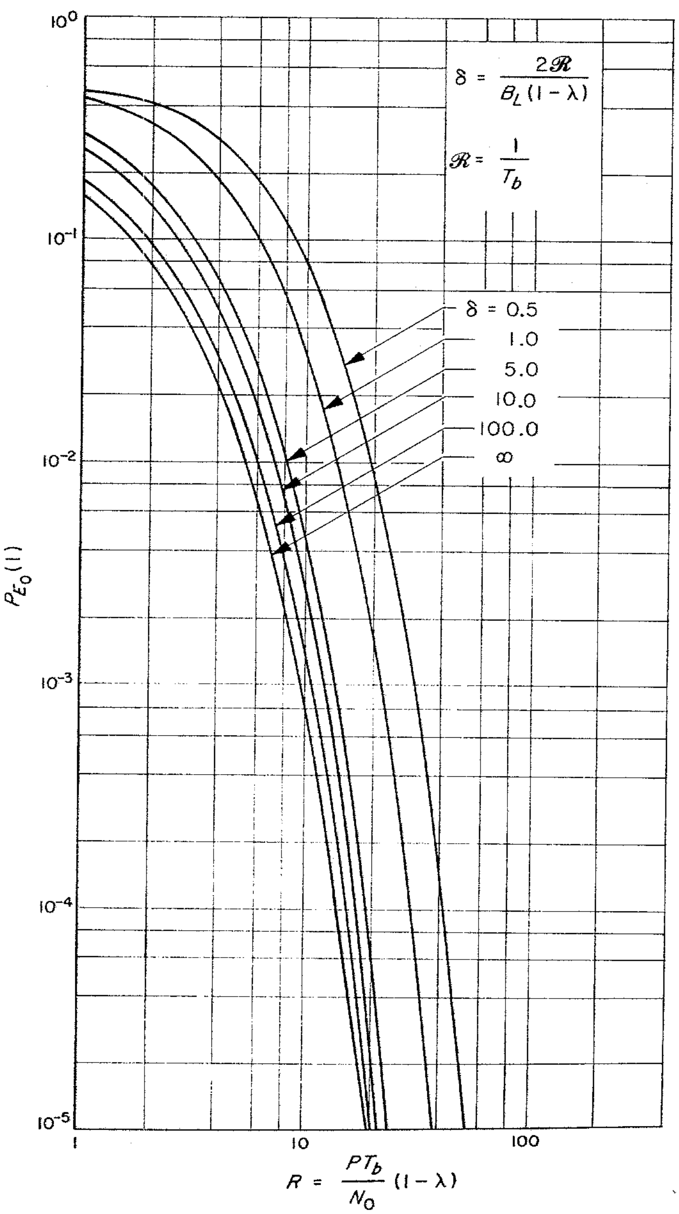

Fig. 2. Error probability $P_{E_0}(1)$ vs R_1 for various values of δ_1

Fig. 3 represents a plot of the parameters R_2 and m_{20}^2, i.e., the value of m_2 which minimizes $P_E(2)$, say $P_{E_0}(2)$. In this case the SNR α_2, in the vehicle carrier-tracking loop, has been set to 9 db. This corresponds to a "near-threshold" condition in the spacecraft's carrier-tracking loop. The results in Fig. 3 have been obtained by fixing R_2 and α_2 and searching, by means of the IBM 7090 computer, for that value of m_2, say $m_{20}^2 = P_{c2}/P_2$, which minimizes $P_E(2)$, say $P_{E_0}(2)$. Finally, Fig. 4 represents a plot of $P_{E_0}(2)$ versus R_2 for various values of the parameter $\delta_2 = 2\mathcal{R}_2/B_{L2}(1 - \lambda_2)$ and $\alpha_1 = 9$ db. Notice that the behavior of $P_{E_0}(2)$ versus m_{20} is similar to that obtained for the one-way link. The major difference is that for large R_2, the $P_{E_0}(2)$ versus m_{20} characteristic exhibits a bottoming (irreducible error probability) behavior which is due to the presence of additive noise on the uplink. The bottoming behavior can be eliminated by using a "clean" carrier reference in the vehicle or by increasing the up-link SNR α_1 to a point where the phase-jitter in

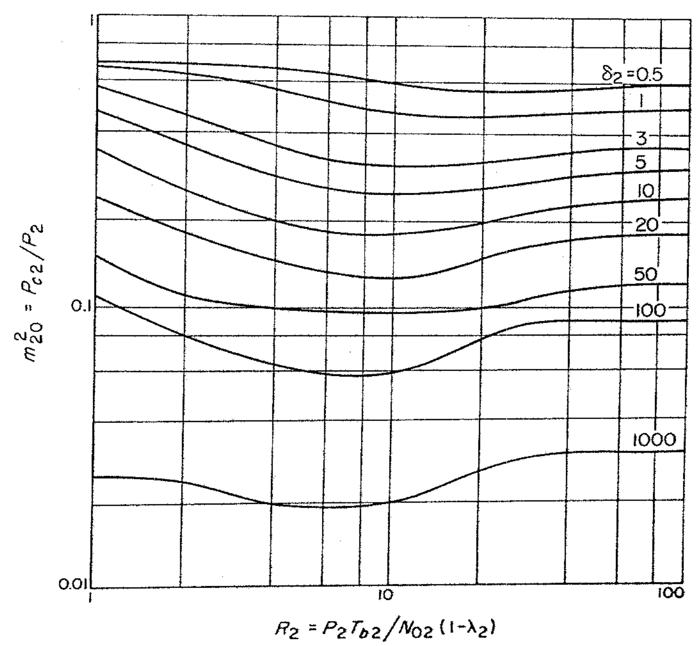

Fig. 3. Optimum modulation index vs R_2 for various values of δ_2 with $\alpha_1 = 9$ db

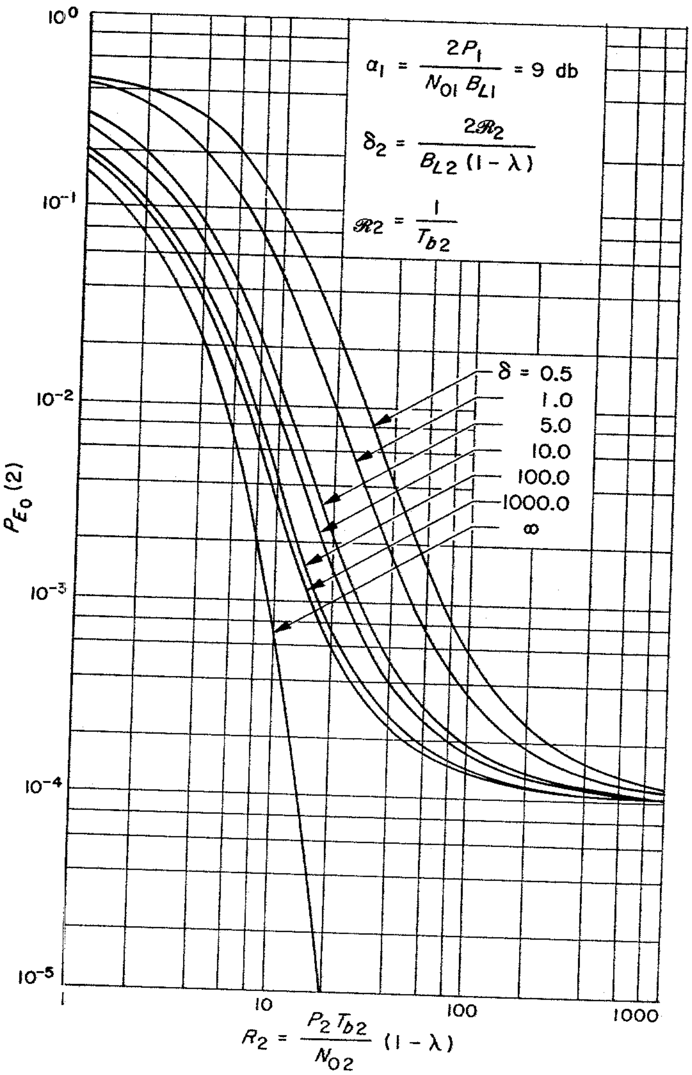

Fig. 4. Error probability $P_{E_0}(2)$ vs R_2 for various values of δ_2 with $\alpha_1 = 9$ db

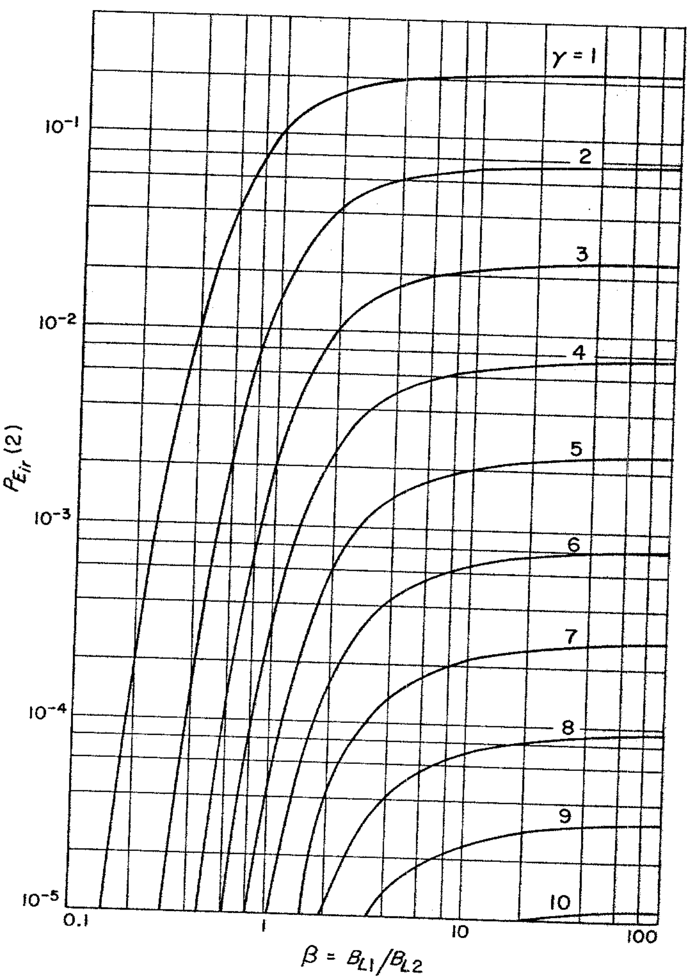

Fig. 5. Irreducible error probability $P_{E_{iR}}(2)$ vs β for various values of $\gamma = 2P_{c1}/N_{01}B_{L1}$

the vehicle's carrier tracking loop becomes negligible. For an error probability $P_E(2) = 10^{-3}$, this occurs for all practical purposes, where $\alpha_1 > 40$ (16 db), *SPS 37-35*, Vol. IV, pp. 339–341.

The irreducible error probability, say $P_{E_{ir}}(2)$, may be obtained from Eq. (3) by letting R_2 approach infinity. In the limit there is

$$P_{E_{iR}}(2) = \frac{1}{2}\left\{1 - \frac{2}{\pi}\sum_{k=0}^{\infty}(-1)^k \varepsilon_k \frac{I_k(\alpha_1)}{I_0(\alpha_1)}\frac{1}{1-4k^2}\right\} \qquad (4)$$

Fig. 5 represents a plot of $P_{E_{ir}}(2)$ versus the SNR $\gamma = 2P_{c1}/N_{01}B_{L1}$ with β as a parameter.

B. An Optimum Squaring Loop Prefilter

J. W. Layland

1. Introduction

Squaring loops have been proposed in the past (Ref. 1) as a means of establishing a coherent carrier reference for 180-deg biphase PSK Modulation. The received signal is bandpass filtered, squared to remove the modulation, and the resultant double-frequency term is tracked by a conventional phase-lock loop (Ref. 1).

Noise, always present in the received signal, is enhanced by the squaring operation. It is, therefore, natural to ask for the presquaring filter, which maximizes the