Appendix G
Simple Approximations: Spacecraft Surface Charging Equations

Whereas Appendix D addresses internal charging analyses, this section will focus on surface charging.

The simple approximations discussed in this section are of a worst-case nature. If this analysis indicates differential potentials between non-circuit surface materials of less than 400 V, there should be no spacecraft discharge problems. If predicted potentials on materials exceed 400 V, the Nascap-2k code (Appendix C.3.3) is to be used.

Although the physics behind the spacecraft charging process is quite complex, the formulation at geosynchronous orbit can be expressed in very simple terms if a Maxwell-Boltzmann distribution is assumed. The fundamental physical process for all spacecraft charging is that of current balance; at equilibrium, all currents sum to zero. The potential at which equilibrium is achieved is the potential difference between the spacecraft and the space plasma ground. In terms of the current [1], the basic equation expressing this current balance for a given surface in an equilibrium situation is:

$$I_E(V) - [I_I(V) + I_{SE}(V) + I_{SI}(V) + I_{BSE}(V) + I_{PH}(V) + I_B(V)] = I_T \qquad \text{(G – 1)}$$

where:

V = spacecraft potential

I_E = incident electron current on spacecraft surface

I_I = incident ion current on spacecraft surface

I_{SE} = secondary electron current due to I_E

I_{SI} = secondary electron current due to I_I

I_{BSE} = backscattered electrons due to I_E

I_{PH} = photoelectron current

I_B = active current sources such as charged particle beams or ion thrusters

I_T = total current to spacecraft (at equilibrium, $I_T = 0$).

For a spherical body and a Maxwell-Boltzmann distribution, the first-order current densities (the current divided by the area over which the current is collected) can be calculated [1] using the following equations (appropriate for small conducting sphere at GEO):

Electrons

$$J_E = J_{E0} \exp(qV/kT_E) \qquad V < 0 \text{ repelled} \tag{G-2}$$

$$J_E = J_{E0} [1 + (qV/kT_E)] \qquad V > 0 \text{ attracted} \tag{G-3}$$

Ions

$$J_I = J_{I0} \exp(-qV/kT_I) \qquad V > 0 \text{ repelled} \tag{G-4}$$

$$J_I = J_{I0} [1 - (qV/kT_I)] \qquad V < 0 \text{ attracted} \tag{G-5}$$

where:

$$J_{E0} = (qN_E/2)(2kT_E/\pi m_E)^{1/2} \tag{G-6}$$

$$J_{I0} = (qN_I/2)(2kT_I/\pi m_I)^{1/2} \tag{G-7}$$

where:

N_E = density of electrons

N_I = density of ions

m_E = mass of electrons

m_I = mass of ions

q = magnitude of the electronic charge.

T_E = temperature of electrons

T_I = temperature of ions

Given these expressions and parameterizing the secondary and backscatter emissions, equation G-1 can be reduced to an analytic expression in terms of the potential at a point. This model, called an analytic probe model, can be stated as follows:

$$A_E \, J_{EO} \, [1 - SE(V,T_E,N_E) - BSE(V,T_E,N_E)]\exp(qV/kT_E)$$

$$- A_I \, J_{I0} \, [1 + SI(V,T_I,N_I)][1 - (qV/kT_I)]$$

$$- A_{PH} \, J_{PHO} \, f(X_m) = I_T = 0 \qquad V < 0 \qquad \text{(G-8)}$$

where:

A_E = electron collection area

J_{EO} = ambient electron current density

A_I = ion collection area

J_{I0} = ambient ion current density

A_{PH} = photoelectron emission area

J_{PHO} = saturation photoelectron flux

BSE,SE,SI =parameterization functions for secondary emission related to backscatter, electrons, and ions

$f(X_m)$ = attenuated solar flux as a function of altitude X_m of center of Sun above the surface of Earth as seen by spacecraft (percent).

This equation is appropriate for a small (<10 m), uniformly conducting spacecraft at geosynchronous orbit in the absence of magnetic field effects. To solve the equation, V is varied until $I_T = 0$. Typical values for aluminum of SI, SE, and BSE are 3, 0.4, and 0.2, respectively. For geosynchronous orbit, J_E/J_I is about 30 during a geomagnetic storm.

As discussed earlier in Eq. (2.3-6), when the spacecraft is in eclipse (and ignoring secondary and backscattered terms), a simple proportionality between the satellite potential and the currents and temperature can be derived from Eq. (G-8):

$$V \sim \frac{-T_E}{q} \times \ln(J_E / J_I) \tag{G-9}$$

where:

T_E is in eV.

That is, to rough order in eclipse, the spacecraft potential is directly proportional to the plasma temperature expressed in electron volts and the natural log of the ratio of the electron and ion currents. Note, however, that secondary currents play a crucial role in actual calculations, and T_E must be greater than some critical value [2–5], usually of the order of 1000 eV, before charging will occur because secondary electron production can exceed the ambient current for low enough T_E. Also, $\ln(J_E/J_I)$ often varies much more rapidly and by larger factors than T_E so that charging has been found often to be more related to changes in $\ln(J_E/J_I)$ than T_E [6].

References

[1] H. B. Garrett, "The Charging of Spacecraft Surfaces," *Reviews of Geophysics and Space Physics,* vol. 19, no. 4, pp. 577–616, November 1981.
A nice summary paper, with numerical examples and many illustrations. This and Whipple (1981) [7] are two definitive papers on the subject, each covering slightly different aspects.

[2] H. B. Garrett and S. E. DeForest, "Analytical Simulation of the Geosynchronous Plasma Environment," *Planetary and Space Science,* vol. 27, pp. 1101–1109, 1979.

[3] R. C. Olsen, "A Threshold Effect for Spacecraft Charging," *Journal of Geophysical Research,* vol. 88, pp. 493–499, January 1,1983.

[4] S. T. Lai and D. J. Della-Rose, "Spacecraft Charging at Geosynchronous Altitudes: New Evidence for Existence of Critical Temperature," *Journal of Spacecraft and Rockets,* vol. 38, pp. 922–928, 2001.

[5] V. A. Davis, M. J. Mandell, and M. F. Thomsen, "Representation of the Measured Geosynchronous Plasma Environment in Spacecraft Charging Calculations," *Journal of Geophysical Research,* vol. 113, no. A10204, doi:10.1029/2008JA013116, 2008.

[6] H. B. Garrett, D. C. Schwank, P. R. Higbie, and D. N. Baker, "Comparison Between the 30-80 keV Electron Channels on ATS-6 and 1976-059A During Conjunction and Application to Spacecraft Charging Prediction," *Journal of Geophysical Research,* vol. 85, no. A3, pp. 1155–1162, 1980.

[7] E. C. Whipple, "Potentials of Surfaces in Space," *Reports on Progress in Physics*, vol. 44, pp. 1197–1250, 1981.
Nice summary paper. Emphasis is on total charging and not internal charging, but good for physics background. This and Garrett (1981) [1] are the two definitive papers on that subject, each covering slightly different aspects.