An Evaluation of Charged Particle Calibration by a Two-Way Dual-Frequency Technique and Alternatives to This Technique

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This article relates to the accuracy of the three charged particle calibration methods – differenced range versus integrated doppler (DRVID), Faraday rotation, and dual frequency – as they apply to the various tracking modes, e.g., one-station tracking, two-station tracking, spacecraft very long baseline interferometry (VLBI). It is found that many calibration schemes are deficient at small Sun–Earth–probe angles (SEPs). Observations of the Sun during its active period between 1967 and 1969 have been used to obtain quantitative information on range degradation at small Sun–Earth-probe angles. Likewise, range errors at SEPs during a quiet Sun period (in this case the 1964–1965 solar minimum) have also been computed with the result that, even at times of a comparatively inactive Sun, range errors engendered by plasma clouds are still troublesome inasmuch as they prevent range measurement with an accuracy of less than 1 meter.

I. Introduction

In order to perform a systems analysis of the best possibilities for calibrating charged particles, the major tracking modes and the three major charged particle calibration methods are presented together with an error analysis.

II. Tracking Modes

The seven major tracking modes can be separated into two general categories: one-station tracking and two-station tracking.

A. One-Station Tracking Modes

(1) *Two-way ranging.* This mode, very well established for years, consists of transmitting a range code toward the spacecraft that, when transmitted back, will be received at the transmitting station and referenced to the range code continuously generated. By this means the round-trip light time is measured and the range ascertained.

(2) *Two-way doppler.* In this case the frequency shift of the RF carrier transmitted and received by a single station is determined and gives information

on the velocity of the spacecraft relative to the station.

B. Two-Station Tracking Modes

In two-station tracking, differences are measured, e.g., differences of range or range rates (doppler) as determined from two stations far apart. As shown in Ref. 1, these tracking modes are used because geocentric range and range rate are subject to a number of errors which, for convenience, are lumped together and termed "process noise" – erratic forces due to solar radiation pressure, pressure leaks in the spacecraft, etc., which are difficult to model. By differencing ranges or range rates as measured from two different stations, the geocentric range will drop out of the relevant equations and most of the process noise will be cancelled.[1] A number of new modes thus are made feasible.

(3) *Two-way minus three-way doppler.* In this mode one station transmits a carrier frequency and two stations receive the returned signal simultaneously. By differencing the information, the range rate difference, $\rho_2 - \rho_3$, is determined.

(4) *Two-way minus three-way ranging.* In this case a wideband signal modulated with a range code is transmitted to the spacecraft from one station, and two stations simultaneously receive the returned signal. By differencing the information, the range difference, $\rho_2 - \rho_3$, is obtained.

(5) *Alternate ranging.* Here, ranging measurements are performed from two stations alternately because two different ranging machines transmit different range codes, and one transponder on the spacecraft cannot transpond the two signals if they are received simultaneously. This technique is operationally complicated, requiring approximately an hour of ranging by one station, then a transfer to the second station for an hour of ranging, then a transfer of the spacecraft to the first station, and so on until the overlap time expires.

(6) *Simultaneous ranging.* This mode of ranging is quite similar to alternate ranging, the only difference being that the two stations range at the same time, which is possible if two frequencies and two transponders are employed. Two frequencies are under active consideration, for reasons to be discussed below.

[1]The reason for this is that the angular motions are much less pronounced than the radial accelerations.

(7) *Spacecraft VLBI.* In this mode of operation two stations are listening simultaneously to the spacecraft. No ground based transmission is involved. The received signals are cross-correlated and the differenced range determined directly.

III. Calibration Modes

Opposite the seven tracking modes are three basic charged particle calibration methods which can be used in any combination.

A. Faraday Rotation (FR)

This calibration method is based on the fact that the plane of a linearly polarized wave will be rotated in a magneto-active plasma. By measuring the rotation, conclusions can be drawn as to the electron content of the intervening plasma. Since the linearly polarized wave may be transmitted from the spacecraft or from an Earth satellite, there exist two different versions of the FR calibration method: satellite FR and spacecraft FR. The inherent accuracies of these two methods are somewhat different and will be discussed later.

B. Differenced Range Versus Integrated Doppler (DRVID)

This calibration is based on the fact that the group and phase velocities of an electromagnetic wave differ in a plasma. However, since the phase velocity is that of a very narrow bandwidth signal (doppler), DRVID is only capable of calibrating range *rates*.

C. Dual-Frequency Calibration

This is by far the most promising method for correcting the range and range rate errors caused by the ionosphere and the interplanetary plasma. It consists of transmitting and receiving the same signal at two different frequencies (S and X band). The difference in integrated doppler and also in range for the two frequencies is a direct measure of the total electron content. The differenced doppler provides a precise but ambiguous charged particle measurement, while the differenced range provides an unambiguous (though noisier) measurement.

IV. Error Analysis of the Major Calibrations

A. Faraday Rotation

The calibrations based on the FR are capable of removing only the range and range rate errors due to the

ionosphere (Ref. 2). This is even true, generally, for the spacecraft FR mode, since the space plasma's magnetic field is very small ($\approx 10^{-6}$ gauss) and the rotation of the plane of polarization amounts only to extremely small fractions of a degree at S-band (10^{-8} rad) while the ionosphere typically produces rotations of the order of 1 to 10 deg of S-band. An exception is the solar corona. When the Sun-Earth-probe angle (SEP) becomes less than 5 deg, the solar plasma activities may give rise to large excursions from the ionospheric background (Ref. 3). Very close to an active Sun, rapid changes of the plane of polarization of 40 deg/h are not uncommon.

An error analysis, (see Appendix A), has been performed on the degrading influence of the solar corona on FR measurements. There it has been shown to expect a range rate error of about 10 m/h due to the solar corona during a sunspot maximum at an SEP of 5 deg. Spacecraft FR calibration will be insensitive to the solar corona for SEPs larger than 5 deg because of the rapid decrease of the magnetic field strength in solar plasma clouds. However, the range uncertainty is still severe ($>$ 1 m). It must be emphasized that the foregoing considerations presuppose an active Sun. During a quiet Sun period, however, the problem of range uncertainties still persists for very small SEPs ($<$ 5 deg) simply because the solar corona is not known as accurately as required. The electron density quoted for instance in Ref. 3 may well be off by a factor of 2 (C. T. Stelzried, private communication). Therefore, for extremely small SEPs the spacecraft FR calibration mode will be uncertain regardless of the status of the Sun's activity.

We can deal with the satellite FR mode briefly, since it has been discussed at length elsewhere (Ref. 4). In short, the rotation of the plane of polarization of an electromagnetic wave in the line of sight between a satellite and an Earth-bound station is measured continuously. In this way the electron content of the ionosphere *in the line of sight between satellite and an Earth station* is determined. However, what is needed for calibration is a knowledge of the electron content in the line of sight between the spacecraft and this station. Therefore the electron content has to be mapped to the line of sight between spacecraft and station. This is done at JPL via the computer program Hyperion. A detailed description of Hyperion is given in Ref. 5. (Certain assumptions, e.g., a Chapman layer for the ionosphere, are inherent to the program.)

Although the above-mentioned assumptions have been proven to be generally correct, the actual deviations from this model are such that a range error of 1.5 m accumulating during an 8-h pass can be expected in summer, and an error of 0.5 m can be expected in fall, winter, and spring. (Ref. 6).

To summarize, both FR modes have certain limitations. The satellite FR mode obviously can calibrate only the ionosphere and must therefore be complemented with other calibration schemes, but may then become quite useful. For the spacecraft FR mode the same holds generally true. Although there is the disadvantage of a lack of true ionospheric calibration at small SEPs ($<$5 deg) during an active Sun period due to the solar corona, this is more than offset by the fact that the previously mentioned mapping with all its uncertainties is unnecessary. Later, when the dual-frequency method has been introduced we will see how the spacecraft FR mode can be used to advantage.

B. Differenced Range Versus Integrated Doppler

This method (see Ref. 7) is used to calibrate for charged particles. Its principle is based on the fact that phase and group velocities differ in a plasma. It can be used to calibrate range rates and range differences only, *never the absolute range*. As far as the accuracy is concerned it must be realized that the main problem in this type of calibration is the ranging channel. The signal-to-noise ratio for the ranging system slated for the outer planets mission is the same as for the Viking mission (G. E. Wood, private communication), and therefore the error analysis by MacDoran (Ref. 8) is applicable. According to this analysis $27 \cdot 10^{-9}$ s "ranging jitter" for a 15-min integration time provides a calibration accuracy to the 1-m level. The hardware requirements for this kind of accuracy do not seem to be insurmountable at all; as a matter of fact Madrid in his tracking system analytical calibration (TSAC) activities (Ref. 9) has already achieved a sigma of 1 m during the Mariner 9 mission. To summarize, DRVID is a useful tool for charged particle calibrations which can confidently be expected to be accurate to the 1-m level, but by its nature cannot calibrate absolute ranges and therefore does not apply to tracking methods 1, 4, 5, 6, or 7.

C. S/X Dual-Frequency Calibration

By far the most promising charged particle calibration mode is the utilization of a dual-frequency system. The question arises immediately whether a downlink only S/X-band system, which is presently planned, is sufficient for an accurate calibration or whether a combined up and down S/X system is needed. An analysis has been

made in Appendix B, showing that at SEPs smaller than 20 deg an uplink *and* downlink dual-frequency system is definitely needed if the Sun is active and the desired accuracy is to be below the 1-m level, the reason being of course that in a time-dependent medium the uplink total electron content cannot be inferred from the downlink content. The analysis has been applied to a numerical estimate of range errors at small SEPs if only a downlink S/X system is used; Fig. 1 gives the results. The solar plasma data used to evaluate Fig. 1 are extracted from Ref. 10 (see also Appendix B).

Summarizing, we emphasize that a dual-frequency calibration, if done on both the uplink and downlink, is by far the most accurate ranging calibration known. If only a downlink S/X band system is used, a range inaccuracy develops at small SEPs (<20 deg). This is particularly true for an active Sun (see Fig. 1).[2]

V. Combination of Calibration Modes

After having delineated the main modes of calibration we will now briefly discuss combinations of them. It is clear that the FR methods can be used only in conjunction with the other two since the former only calibrates the ionosphere. On the other hand the two-way dual-frequency method is self sufficient. A promising combination of methods is combining either of the FR modes together with the downlink-only dual-frequency method. For, if the space plasma is quiet, because of a quiet Sun or because the SEP is large, the only long-term time variations are the diurnal variations of the ionosphere. The calibration for the uplink is then provided for by FR. If the space plasma is active, the downlink dual-frequency measurement will differ from the FR measurement and, though the exact calibration cannot be computed, the size of the error caused by neglecting the uplink space plasma effect can be bounded. It appears then that the most promising charged particle calibration techniques are:

(1) Uplink and downlink dual-frequency.

(2) Faraday rotation plus downlink dual-frequency.

Whereas the first method is foolproof, the second has some limitations. These limitations relate to the fact that for small SEPs and an active Sun, range uncertainties will occur as depicted in Fig. 1.

[2]For details see Appendix B.

One interesting combination, however, exists in which DRVID may help the downlink-only S/X-band *ranging* capability. Suppose the solar plasma is quiescent for some time prior to $t = t_0$. The downlink electron content is then the same as the uplink content, and the uplink content can therefore be determined. Suppose further that at t_0 and thereafter, solar plasma clouds and streamers enter the ray path and change the electron content rapidly (changes of $5 \cdot 10^{17} m^{-2}$ in the electron content within 30 min are not uncommon). DRVID will immediately become active and the uplink electron content may be determined. However, for SEPs less than 20 deg occasions of this type are rather rare.

VI. Comparison of Techniques

We can now compare the charged particle calibration modes and cross-correlate them with the various tracking modes mentioned at the beginning of this article. This is done in Table 1. The various ranging and tracking modes are listed according to the number assigned to them in Section I. It is to be noted from Table 1 that spacecraft VLBI is adequately calibrated with a downlink frequency system only. This is of course obvious, but it should be pointed out specifically. On the other hand, it would seem that the *differenced* tracking modes (3 to 6) are not beset by the range calibration uncertainties for small SEPs of the downlink-only S/X-band calibration method, since the two ray paths, separated by some 7000 km, experience generally the same plasma activities, and when the ranges are differenced, the uncertainties cancel to the 10-cm level. This happens indeed to be true for modes 3 and 4. Both of these modes have one common uplink, and therefore any range degradation caused by plasma clouds passing the ray path during the transit of the radio signal on its uncalibrated uplink will exactly cancel upon differencing the ranges. However, for modes 5 and 6 this is not true. In mode 5 the two stations track about an hour each alternately, and the uncalibrated uplink gives trouble at small SEPs since plasma clouds can come and go within an order of hours. Finally, mode 6 cannot be calibrated properly with a downlink-only S/X-band system simply because the two stations operate at two different frequencies and the space plasma affects them differently.

VII. Summary

We have described three methods by which the degrading influence of electromagnetic plasma interactions on range and range rate values can be eliminated, at least partially. Table 1 gives the results of this analysis.

We have found that the S/X-band dual-frequency calibration, if employed for both the uplink and downlink, is superior to all other methods, particularly for one-station tracking. However, downlink-only S/X is quite adequate for two-station tracking in modes 3, 4, and 7. We have also seen that a combination of Faraday rotation with downlink-only dual-frequency is a viable candidate for charged particle calibration. Both methods work equally well during times of a quiet Sun; but at small Sun–Earth–probe angles and during periods of an active Sun, range calibration errors for downlink-only dual-frequency may become severe (see Fig. 1).

Table 1. Expected total range error over one pass due to charged particle calibration uncertainties

Ranging or tracking			Charged particle calibration mode				
			Satellite FR[a]	Spacecraft FR[b]	DRVID	S/X, downlink only	S/X, uplink and downlink[c]
ρ_2	(1)	(1)	1.5 m	1.5 m	Not applicable	0.5 m[d]	0.5 m
ρ_2	(2)	(2)	1.5 m	1.5 m	1 m	0.5 m[d]	0.5 m
$\rho_2 - \rho_3$	(3)	(3)[e]	2.0 m	2.0 m	1 m	0.5 m[f]	0.5 m
$\rho_2 - \rho_3$	(4)	(4)[e]	2.0 m	2.0 m	Not applicable	0.5 m[f]	0.5 m
Altitude ranging		(5)[e]	2.0 m	2.0 m	Not applicable	0.5 m[e]	0.5 m
Simultaneous ranging		(6)[e]	2.0 m	2.0 m	Not applicable	0.5 m[e]	0.5 m
Spacecraft VLBI		(7)[e]	2.0 m	2.0 m	Not applicable	0.5 m[f]	Not applicable

[a]Calibrates the ionosphere only.

[b]Calibrates the ionosphere only except for Sun–Earth–probe angles less than 5 deg, when the corona degrades calibration to an unacceptable level.

[c]In this ideal mode only instrumentation limitations are present. They are estimated to be at the half-meter level.

[d]The hardware limited value quoted is only applicable if the Sun–Earth–probe angle is larger than 20 deg and the Sun is not active (Fig. 1).

[e]The reason for the increase in inaccuracy for tracking modes (3) to (7), relating to the Faraday rotation, is the fact that the ionospheric environment differs between the two tracking sites and an rms average was taken.

[f]The only tracking modes for which downlink *only* S/X-band is totally adequate.

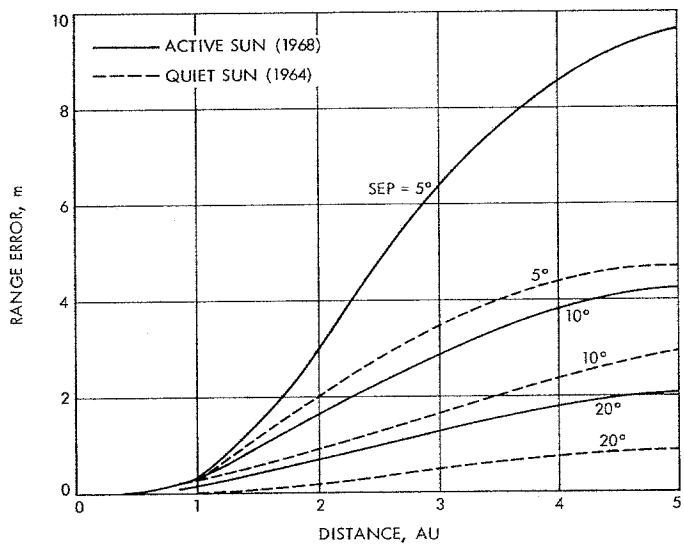

Fig. 1. Estimated range error for S/X-band downlink calibration only, due to time variation in the solar wind (valid for one-station two-way ranging ρ_2)

Appendix A

Range Degradation Due to the Solar Corona

The following analysis gives an estimate of the degrading influence of the solar corona on FR measurements. To be specific let the SEP be 5 deg, corresponding to a closest distance of the ray path from the Sun $R = 10$ (measured in Sun radii). Let us take the improved plasma electron density as given by Stelzried (Ref. 3):

$$N = 10^{14}\left(\frac{6000}{R^{10}} + \frac{0.002}{R^2}\right)(\text{in m}^{-3})\ (4 < R < 12) \qquad \text{(A-1)}$$

This is valid for the quiet Sun. If the closest distance of the straight ray path from the Sun is R_1, then the total electron content within the sphere of the influence of the solar corona ($R < 12$) for a quiet Sun is given by

$$I = 10^{14} R' \int_{-(12^2 - R_1^2)^{1/2}}^{(12^2 - R_1^2)^{1/2}} \left(\frac{6000}{(R_1^2 + x^2)^5} + \frac{0.002}{R_1^2 + x^2}\right) dx\ (\text{in m}^{-2}) \qquad \text{(A-2)}$$

where R' is the radius of the Sun in meters. Now, for $R_1 = 10$ this amounts to

$$I_{10} = 5.6 \cdot 10^{18}\text{m}^{-2}$$

corresponding to about 36 m of range error at S-band. Adopting a Parker magnetic field, the Faraday rotation turns out to be some 10 deg. When plasma bursts occur from an active Sun, similar polarization changes (10 deg/h) have been observed on Pioneer 6 (Ref. 3), which would indicate that range rate errors of some 30 m/h can occur under the assumption that the same magnetic fields prevail (this is for an SEP of 5 deg). However, the magnetic fields in plasma clouds close to the Sun are likely to be larger by a factor of 2 to 10, which cuts down the range error by the same factor. We therefore expect a range rate error of about 10 m/h due to the solar corona during a sunspot maximum at an SEP of 5 deg.

Appendix B

Analysis of a Time Changing Solar Plasma

The following analysis will show that at SEPs smaller than 20 deg, an uplink *and* downlink dual-frequency system is definitely needed if the Sun is active and the accuracy derived is to be below the 1-m level[3]. The analysis goes as follows: Let R_s be the range as seen at the S-band frequency (Ref. 11)

$$R_s = R + \frac{\alpha}{\omega_s^2} \int_0^R N\left(x, \frac{x}{c} + t\right) dx + \frac{\alpha}{\omega_{st}^2} \int_0^R N\left(x, \frac{2R - x}{c} + t\right) dx \tag{B-1}$$

where

$$\alpha = \frac{\pi e^2}{m}$$

ω_s = S-band frequency

ω_{st} = S-band transponder frequency

R = true range

$N(x,t)$ = electron number density as a function of ray path x and time t

On the other hand the range as seen at the X-band frequency is, according to the foregoing, given by

$$R_x = R + \frac{\alpha}{\omega_x^2} \int_0^R N\left(x, \frac{x}{c} + t\right) dx + \frac{\alpha}{\omega_x^2} \int_0^R N\left(x, \frac{2R - x}{c} + t\right) dx \tag{B-2}$$

where

ω_x = X-band frequency.

Were it not for the explicit time dependence of the electron density, a time dependence which is particularly annoying and unpredictable when the ray path passes near the Sun, Eqs. (B-1) and (B-2) could easily be solved for the two unknown quantities R and the electron content. However in reality we have, in general, two equations and three unknown quantities.

To extract information on R proves to be generally impossible. Since the range is measured at many different times, the only possibility might be to shift the time by an amount Δ such that

$$\int_0^R N\left(x, \frac{x}{c} + t\right) dx = \int_0^R N\left(x, \frac{2R - x}{c} + t + \Delta\right) dx \tag{B-3}$$

The unknown integrals of Eqs. (B-1) and (B-2) are determined by differencing (B-1 and (B-2). In general, no such Δ exists, and therefore a complete charged particle calibration is not possible. If, however, the time-dependent activity of the solar wind is localized in the path at x_0, say, we may represent the electron density by

$$N = L\delta(x - x_0)\, F(t) \tag{B-4}$$

where L is the special extent, and have from Eq. (B-4),

$$F\left(\frac{x_0}{c} + t\right) = F\left(\frac{2R - x_0}{c} + t + \Delta\right) \tag{B-5}$$

so that $\Delta = 2(x_0 - R)/c$ indeed satisfies Eq. (B-4). However, we do not know x_0.

As to the numerical estimates underlying Fig. 1, we notice first that the electron density behaves approximately as R^{-2} and the path length through a plasma cloud goes as R, where R is the distance from the Sun

[3]The considerations given here are applicable for tracking modes 1, 2, 5, and 6. In tracking modes 3 and 4 the space plasma is "differenced away" to well below the 1-m level.

(see in particular V. V. Vitkevich, Ref. 10), so that the total electron content of a plasma cloud is expected to behave as:

$$\int N ds \sim \frac{1}{R} \sim (\mathrm{SEP})^{-1} \tag{B-7}$$

Furthermore at an SEP of 20 deg the distance from the Sun is sufficiently large so that the electron content time variations as found by the Stanford group (R. L. Koehler, T. A.Croft, Ref. 10) apply. The two integrals in Eqs. (B-1) and (B-2) are expected to differ by 2 m then in the limit of large R. For intermediate R the time delay between uplink and downlink becomes shorter and the difference between the two integrals in Eqs. (B-1) and (B-2) becomes progressively smaller. The references quoted so far pertain to an active Sun period. During a quiet Sun period there is still sufficient activity (plasma clouds, flares, etc.), however, to degrade the range determination by a few meters at small SEPs (Ref. 12).

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